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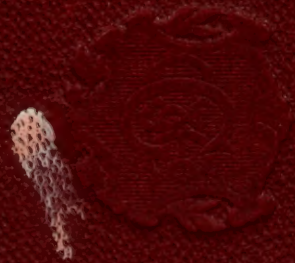

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BY

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"Probable evidence, in its very nature, affords but an imperfect kind of information. . . . But to us probability is the very guide of life."

JOSEPH BUTLER, D.C.L.,
Bishop of Durham.

Printed in Great Britain.
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PREFACE.

THE object of the present work is to lay before mining engineers and others interested in the subject of which it treats my views upon the principles underlying Mineral Valuation, in the form into which they have slowly crystallised during the course of many years of experience in the practice of this branch of the mining profession. Although I am taking for granted a general acquaintance with mining practice, I have, nevertheless, included in the book certain matters that will appear elementary to the experienced mining engineer, with the desire of making the work intelligible to those outside the mining profession, as well as to students just entering upon their career. I am concerned to bring out in the first instance the cardinal fact that all mineral valuations must be based upon an estimate of probabilities, never of certainties, and that, in strict accordance with the laws of probability, every now and again cases will occur in which the result will depart more or less widely from that which the known data would indicate as the most probable. This consideration is too often overlooked by those who have to do with mineral valuation, and perhaps more particularly by those who are in the habit of dealing with the conclusions derived therefrom. This does not mean that mineral valuation must be looked upon as an abnormally un-

reliable operation, for it must not be forgotten that a similar element of probability runs through all mundane affairs and guides all our actions, though we are too often apt to forget that what we call certainties are merely probabilities of a very high order. I have endeavoured to develop this theme and to show how the mathematical conceptions of probability can be applied to mining valuations, so as to give a more definite meaning to the results obtained by them, and can only add that in my own practice the applications of these principles have stood well the test of long experience.

It will be noted that this work contains no Tables, such as are to be found in most books on valuation. I have omitted these for a number of reasons: For one thing, there are plenty of good Actuarial Tables available for those who wish to make use of them. It is, however, impossible to provide Tables corresponding to all possible rates of interest, and I have noticed that those who rely wholly on Tables insensibly get into the habit of using those rates of interest and those conditions which they find in their Tables, rather than the data which most accurately fit the case before them. My own practice for many years has been to make the calculations in every case for myself, and the task is by no means an onerous one. The collection of the relevant data to enable a mineral valuation to be prepared is the work of weeks, sometimes of months, whilst it will be an exceptionally intricate calculation that takes an hour to perform, and there seems little object under these circumstances in endeavouring to

save, say, ten minutes by using Tables. It must not be forgotten that in any case, whatever tables are used, the necessity of making some calculations will always remain. It will be seen that I have throughout endeavoured to get the formulas which I use into the form best adapted to logarithmic computation, although I have also indicated how Actuarial Tables can be used to shorten the work whenever there is a likelihood of their proving helpful. On the other hand, I recommend the use of five-figure logarithms for general practice, instead of the more precise tables usually employed by actuaries, whose work consists in dealing with exactly ascertainable figures. When the data upon which a valuation is based are always necessarily affected by probable errors, generally considerable and often very large, meticulous refinement in precise calculation is absolutely useless and a sheer waste of time. Insistence on such precision always reminds me of one of my students, who was told to calculate the number of bricks required to line a shaft of given depth and diameter, and who worked out the result to 0·001 of a brick !

I venture to think that, as far as the actuarial side of valuation is concerned, the formulas which I am giving will be found to answer all purposes ; on some of the points involved there is still room for differences of opinion between valuers, and I have endeavoured to do justice to all of these : whilst being very far from wishing to assert dogmatically that the methods which I myself use are the only correct ones, I have naturally indicated those to which I give the preference, and I

have set out as far as possible the reasons that have induced me to adopt the methods which I advocate here. Apart, however, from the actuarial aspect of valuation, there remains the larger and more difficult portion of the problem, which consists in collecting the data upon which the calculations are based, and this portion demands the highest qualities of the trained mining engineer with a diversified mining experience and a thorough knowledge of mining geology. For this side of the valuer's work neither rule can be given nor formula devised ; experience is the only safe guide, and the capacity for taking infinite pains the most important qualification.

HENRY LOUIS.

NEWCASTLE-ON-TYNE,
April, 1923.

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MINERAL VALUATION.

CHAPTER I.

GENERAL CONSIDERATIONS.

The valuation of any article means the process of determining the value thereof. Most writers on Economics have attempted, though without any conspicuous measure of success, to frame a satisfactory definition of value. For the most part they are agreed that it represents a ratio—namely, the proportion which the article valued bears to the standard with which it is compared, and in this sense it is generally held that there is no such thing as an intrinsic value. If, however, the standard of comparison is regarded as having a fixed value, the term intrinsic value may fairly be employed to designate the ratio relatively fixed for the time being between the article and the standard, due to the inherent qualities of the article valued, and the phrase will be employed strictly in this qualified sense wherever it is used in the present work. Economists distinguish between value in use and value in exchange, the latter being the sense which is generally implied when the word value is used without any qualification. It will be seen subsequently that in a few of the cases to be considered, the value in exchange is based upon the value in use, although the instances of this are somewhat exceptional. It is generally admitted that when the value in exchange represents the

ratio of value to a monetary standard, value is practically synonymous with price, so that under these conditions the valuation of an article is the operation of determining its equitable price. It would appear, therefore, that all the work of modern economic studies carries the subject very little further than the old canon of Roman Law :—"*Res tanti valet quanti vendi potest*" —"A thing is worth as much as it can be sold for." This method of valuing an article by what it will fetch in a free market is applicable to the great majority of ordinary commercial commodities ; it can be applied for example, to machinery or plant or stocks of severed minerals. A merchant in the habit of dealing with these can assess with tolerable accuracy the fair market price as between a willing seller and a willing buyer, and in the case of many minerals, market quotations are even published regularly. The method fails, however, in respect of the articles that form the objects of mineral valuation ; in these there is no such thing as a public market, there are no merchants dealing in them regularly, and there are no such things as market quotations. Another basis of valuation has accordingly to be sought, and this may be found, to use the words of Friedrich von Wieser,* in "the expected money proceeds or the exchange value of that money." These expected money proceeds are in the case of mineral valuations not produced in one lump sum, but are spread over a number of years, and it is their capitalised value that has to be determined, so that mineral valuation consists in determining the sum of money which, paid forthwith, is the true equivalent of the money proceeds which are expected to be regularly produced throughout a given period of time by the object valued.

* "Natural Value," by Friedrich von Wieser, Trans. by C. A. Malloch (1893), p. 48.

The mineral valuer has to deal with two essentially different forms of property—namely, mineral deposits, which may be in any stage of discovery or development, and mines opened upon such mineral deposits, such mines being going concerns, which may be at any point in their career of activity. Occasionally both the mineral deposit and the mine may be combined under one ownership, and the two may then be treated as one single asset, but it more often happens that the ownership is divided, and that each must be looked upon as entirely independent of the other. By mineral valuation is accordingly meant the determination of the equivalent money value of the property—that is, the sum that may equitably be given in exchange for it so that neither party to the transaction shall be either the richer or the poorer thereby ; the intrinsic value may be either greater or less than the realisable price, for the time being, of the asset, such price depending upon many conditions besides that of absolute value. Furthermore, it will be shown in the sequel that it is impossible to assign to any mineral property an absolutely definite value, but only a most probable value—that is to say, a value that shall be nearer to the true intrinsic value than any other that can be assigned to it, so that, although the value thus calculated should always form the basis for determining the selling price of the property in question, the actual price realised will depend upon many other conditions, and above all upon the respective needs of the seller and the purchaser—that is to say, whether the desire of the owner to sell is greater than that of the purchaser to acquire, or *vice versâ*. There may also be a special value in position, in that a mineral property, relatively unimportant in itself, may form the means of access to other mineral properties, and on that account may

have a value for one or other party far in excess of its intrinsic value. Accordingly, even after the mineral valuer has assigned a value to a property, the actual purchase price will always in the last instance be a matter of bargain between the interested parties, with the final settlement of which the mineral valuer has little or no concern.

The definition of what constitutes a mineral in the commercial sense in which the word is here used is one that has repeatedly been the subject of legal decisions. It may fairly be defined as a constituent of the Earth's crust, which possesses in its severed condition an intrinsic value by reason of its composition or properties. Lord Justice Fletcher Moulton * has defined a mineral as "any substance that can be got from within the surface of the Earth which possesses a value in use apart from its mere possession of bulk and weight," and Mr. Justice Buckley † has said that "a mineral *prima facie* includes anything lying in the land which has a value of its own as being capable of being used independently of the land." All these definitions, however, are necessarily governed by the context of the document in which the word occurs, and they are subject to the often quoted dictum of Mr. Justice James ‡ supported by Lord Halsbury § that, "in a grant of minerals," it "is a question of fact what these words meant in the vernacular of the mining world, the commercial world, and land owners at the time that they were used in the instrument."

The definition of a mineral deposit offers still greater

* Great Western Railway Co. *v.* Carpalla, etc., Co., Court of Appeal, November 23rd, 1908.

† Great Western Railway Co. *v.* Blades, Chancery, 1901.

‡ Hext *v.* Gill, Chancery, 1872.

§ Lord Provost of Glasgow *v.* Fairie, Court of Appeal, 1889.

difficulties ; it may perhaps be sufficiently described as a portion of the Earth's crust that contains a mineral or minerals as above defined in a sufficiently concentrated form to be capable of economic exploitation under favourable conditions. Thus under exceptional circumstances a gravel containing as little as three grains of gold to the cubic yard, or approximately one part by weight in six millions, is workable to a profit under favourable conditions, and may, therefore, legitimately be classed as a mineral deposit. On the other hand, although ordinary clay contains at least 5 per cent. of iron, it could never be used economically for the production of iron, and hence in this sense would not be called a deposit of iron ore. Lord Herschel * used the phrase "all such bodies of mineral substance lying together in beds, seams, or strata as are commonly worked for profit and have a value independent of the surface of the land," and this is a fairly clear definition of a mineral deposit, except that the words "beds, seams, or strata" do not by any means cover all types of deposits.

The word "mine" is used in this work to include any artificial excavation or working made for the purpose of exploiting a mineral deposit as above defined ; thus it would include oil wells and brine wells as well as quarries. The object of the working is, however, an essential element of the definition ; a tunnel made for the purpose of a railway is thus not a mine, though an exactly similar adit to open up a coal seam is ; an ironstone quarry would be included in the term mine, but borrow-pits made to supply material for embankments would not. It may be added that the distinction between a mine and a quarry is a purely legislative one ; in this country a mine is defined by law as a

* Lord Provost of Glasgow *v.* Fairie (1889), 13 A.C. 657.

working for mineral carried on by artificial light without intentionally disturbing the surface, whilst a quarry is an excavation for the extraction of mineral which is open to the day.* In the United States of America the distinction is based on the nature of the mineral wrought; thus a working for ironstone, whether subterranean or open-cast, is classed as a mine, whilst a working for slate, whether open to the day or not, ranks as a quarry.

Furthermore, in addition to the excavation proper, the term "mine" here also covers the appliances, plant, and equipment used directly or indirectly for the exploitation of mineral, as well as that required for preparing the latter for the market; in some cases it may even be extended to include aerial ropeways, mineral railways, sidings, etc., used for the transport of the mineral from the workings to the spot where it is placed upon a public railway or to the place of shipment from which the mineral is sent away for sale.

The valuation of these mineral assets implies generally the determination of a sum of money such as will produce a total revenue equal to the probable total income which is expected to be yielded by the former. It is, therefore, necessary in the first instance to collect all the relevant data that may enable the amount of such income to be determined. Obviously these must include the quantity of mineral expected to be wrought, the length of time over which such working may be expected to continue, the estimated annual production, the cost of such production, and the probable market value of the product. It need hardly be said that experience is the only guide that the mining engineer can rely upon in order to enable him to estimate these items

* For various definitions of a quarry, see The Quarry (Fencing) Act, 1887 (Vic. 50 & 51, C. 19), The Quarries Act, 1894 (Vic. 57 & 58, C. 42), and The Factory and Workshops Act, 1901 (Ed. VII. 1, C. 22).

with any reasonable measure of accuracy, and even under the most favourable conditions a fairly close approximation to the actual results is all that can be expected. Necessarily much depends upon the state of development of the property to be valued. It is clear that a mine that has been worked for a number of years will provide a far more accurate basis for valuation than a mineral deposit, the existence of which has only just been proved. When the net annual income to be expected from the mineral property and the period of time over which it may be expected to yield such annual income have been determined, the valuation resolves itself into finding the capital sum that would yield such income, allowing, of course, for the replacement of capital at the end of the period in question. In other words, the income is treated as an annuity, and the present value of this annuity has to be determined, this being the value to be placed upon the mineral property. In making this calculation, once the data have been determined, the only difficulty is that of assigning a suitable rate of interest which the capital invested should be allowed to earn. This rate must necessarily be greater than the normal rate of interest upon gilt-edged securities, because the operation is a far more risky one than simple investment in such securities, and the investor in mineral properties naturally requires compensation for the risks he is running as an inducement to enter upon so speculative an enterprise. If it were possible in any particular class of mining operations to determine the ratio of successes to failures, it would be a relatively easy matter to calculate the proper rate of interest to be allowed. For instance, if it were known that in a certain class of mining enterprises, two out of three turned out to be failures, paying only such dividends as would just allow the capital originally

invested to be replaced at the end of the life of the mineral property, the rate of interest to be demanded would be three times the gilt-edged rate plus so much as would replace the capital invested in the third mine at the end of the period, for in this way the result to the investor who invested an equal amount in each of the three properties would be the same in the long run as if he had invested in gilt-edged securities. Again, if two out of the three investments were complete failures involving the total loss of the capital invested in them, a still higher rate would be required, so as to enable the one successful investment to pay the interest upon the capital invested in the two unsuccessful affairs as well as to replace the capital sunk in all three ventures. If the gilt-edged rate were 5 per cent., and the life of all the properties were 20 years, it would require approximately £3 per annum to amount to £100 at the end of that period, so that on the first hypothesis the interest to be required would be 18 per cent., and on the second hypothesis 24 per cent., in order to make the speculation equivalent to an investment in gilt-edged securities. For the sake of simplicity the question of income tax has not been taken into account in the above statement, though it obviously must enter into all complete calculations of this kind, and in practice would considerably increase the rate of interest that ought to be yielded by the mining investment. Unfortunately, it is not possible to obtain in fact any such precise data ; it is difficult, though in some cases possible, to determine approximately the interest yielded upon the capital invested in a certain class of mines as long as these mines are in operation, but this figure takes no account of the many total losses in unsuccessful exploratory work, and in mines that have been complete failures, and have accordingly passed out of existence. In dealing with

this aspect of the problem it is, therefore, only possible to work upon estimates, and these at best can only be approximations to the actual facts. The rate of interest to be allowed is probably the item in which the greatest divergence of opinion is to be expected amongst mining engineers, and it is certainly the one that presents the most difficult portion of the problem in any valuation.

In valuing any mineral property, the nature of the tenure of such property is a matter of prime importance, and it is always necessary to investigate the ownership of a mineral deposit or of the particular portion thereof which is under consideration. There are two entirely different principles of mineral tenure; in one the mineral is or becomes the absolute property of an individual; in the other it remains the property of the State, which concedes the right of working it to individuals for a period of time that may be either limited or unlimited, in return for the payment of certain dues, which as a rule are relatively low. The former method is in force in Great Britain and the United States of America, the latter in France, Germany, Spain, Scandinavia, and indeed in most other countries.

In Great Britain successive legal decisions have established as a matter of fact that the ownership of mineral (with the exception of gold and silver in England) is vested in the owner of the surface, with a few exceptions, of which the Forest of Dean is the most noteworthy, although even there the ownership of the minerals by the Crown is based on the same principle. Under-sea minerals belong to the Crown, but that is also by virtue of the principle above stated, because the Crown claims the ownership of the foreshore and of the sea-bed. Of course, the original or any succeeding owner of the minerals had power to deal with them as

he saw fit, and could, therefore, alienate them or any part thereof whilst retaining the ownership of the surface, so that it is quite common to find possession of the surface and of the minerals under that surface vested in different hands. An owner of minerals may himself work them or propose to work them; in this case it is generally more convenient to consider the profits derivable from the ownership of the minerals and those obtained from their exploitation as merging into one common profit, and to value that as one single unit; it is not suggested that these profits are not essentially different in their nature, for it cannot be denied that they are, but for most purposes the valuation is greatly simplified by looking upon the ownership of the minerals as merely increasing the profit produced by working them.

It is, however, more usual to find that the owner of the minerals leases them to a lessee, either an individual or a corporation, who works them. In this case there are two independent interests that may become the objects of a valuation—namely, the interest of the mineral owner in the revenue derived from the mineral lease and the interest of the miner in the profits produced by the exploitation of the minerals. It must be borne in mind that a mineral lease is not really a lease in the proper acceptation of the term, which strictly means a covenant to allow to another the usufruct of an asset, but is in reality an agreement for sale. When a man rents a house the essential condition is that the lessee shall have the use of the house during the term agreed on, but shall restore it to the lessor at the end of such term practically in the same condition as that in which he received it. In a mineral lease, on the other hand, it is an essential condition that the lessee diminishes or destroys the asset that forms the subject of the

contract. Thus Henn Collins,* Master of the Rolls, says that "a mining lease is really a sale payable by instalments." For income tax purposes, however, the sum received for the sale outright of a mineral property is regarded merely as a change in the nature of the capital, and is not liable to taxation, whereas the revenue derived from a mineral lease is thus liable, although this revenue is in fact compensation for the actual diminution in the capital value of a wasting asset. Mineral leases usually stipulate for the payment of a royalty of a certain amount per ton of mineral extracted or disposed of; other methods of payment are met with, as, for example, a certain sum per acre of coal seam of given thickness wrought out, but it will be seen that this really amounts to a tonnage basis. Most leases further stipulate for a certain rent or dead rent, an annual amount that has to be paid whether any mineral is worked or not; such dead rent may either be an additional rent—that is to say, that it has to be paid over and above the royalty on the mineral wrought, or it may be a merging dead rent—that is to say, that in return for it the lessee is entitled to work without further payment as much mineral as would upon the agreed royalty bring in a sum equal to the stipulated dead rent. The latter is perhaps the more usual method of the two. In colliery leases it is quite usual to allow the lessee a certain amount of coal for colliery consumption upon which no royalty is payable. In valuing a mine which is working upon leasehold minerals, one essential consideration is whether the life of the mineral deposit is estimated to exceed or to fall short of the term of the lease; in the former case it is the termination of the lease, in the latter case it is the exhaustion of the deposit which determines the period of time which the valuer

* In *Re* Aldam's Settled Estates, Chancery 2, 1902.

must consider in his calculations. Some leases contain a clause to the effect that the lease is renewable at the option of the lessee, in which case the latter is entitled to take credit for the renewal period as well as for the term of the original lease. In connection with merging dead rents, it is often stipulated that if the amount of mineral worked in any one year falls short of that corresponding on the agreed royalty basis to the dead rent payable in that year, the lessee shall be entitled to work in a subsequent year free of royalty so much mineral as corresponds to the quantity by which in a previous year he fell short of the tonnage corresponding to the dead rent. This right to make up short workings, or to work "shorts", as it is often called, is sometimes limited to a definite number of years, or sometimes extends throughout the whole period up to the expiration of the lease. The valuation of this right is always a troublesome problem; it is certainly a contingent asset, but it is generally very difficult to determine the extent to which it may be possible to exercise this right, and hence to assign to it a correct monetary value (see Chaps. IV. and VII.).

The absolute ownership of minerals is not the only form of such ownership; cases of limited ownership also occur, the most important being tenancy for life and copy-hold. A tenant for life of an estate has the right of temporary possession during his lifetime of the sub-soil and of the minerals contained therein, but the reversioner has the right of property by inheritance in these same minerals; generally speaking, a tenant for life has a right to work mines which were opened when he entered into possession and to dispose freely of the minerals extracted, and he can give a valid lease of such minerals; he can, of course, never convey any greater rights than those which he himself possesses. In many

cases he has not the right to open up mineral deposits hitherto unwrought, and can, therefore, grant no valid lease of such deposits. Necessarily a valuation of his minerals can extend only to such mines or minerals as he is entitled to work or to dispose of, so that the valuer must in such cases ascertain precisely what the rights of the tenant in fact are. The period over which such rights extend is generally taken to be the expectancy of life of the tenant at the time of the valuation, and this can be determined by reference to any of the accepted mortality tables.

Under the Settled Land Act of 1882 (Vic. 45 and 46, C. 46) a tenant for life of a manorial estate may sell the free-hold or the inheritance of the copy-hold within the manor with or without reservation of all or any mines or any rights or powers relating to mining purposes so as to effect an enfranchisement. He may also lease any mines in the settled lands up to a term of 60 years, and he has very wide powers in determining the form of such lease. The rights of the tenant under this Act are thus defined with considerable accuracy.

In copy-hold lands the property in the minerals is generally vested in the lord, whilst the copy-holder has them in possession. In such cases neither party can work the minerals without the consent of the other. By enfranchisement the copy-holder's title can be converted into a free-hold, and in some cases he may thus acquire the ownership of the minerals. This is not necessarily the case when the enfranchisement is subject to the Copy-hold Act of 1894 (Vic. 57 & 58, C. 46), which stipulates that the property of either the lord or the copy-holder in the mines or minerals shall not be affected by such enfranchisement without the written consent of the tenant or the lord as the case may be, and the price to be paid for such consent may either

be a matter of bargain or may be referred to an arbitrator. It is, therefore, almost impossible to state definitely what share either of the parties has in the minerals or to place a definite value upon such share. As, however, the interest of neither party is greater than that of the other, it will as a general rule be equitable to assign one equal half of the value to each whenever it is necessary to value the lord's or the copyholder's interest in the minerals.

In countries where the concession system obtains the position is entirely different. No individual can own minerals, which are and remain the property of the State for all time, as long as they remain in their unsevered condition. The State, however, grants concessions—that is to say, it concedes the right to work these minerals and to dispose of them, the ownership of the minerals after severance being in the consessionaire. As a general rule, such concessions are in perpetuity or until the exhaustion of the mineral deposits, and are subject to the payment of annual dues which are generally not very heavy. The concessionaire has thus many of the rights of an owner in fee simple, subject to the payment of the stipulated dues to the State; occasionally there are, however, certain other conditions to be observed, and in many countries the concessionaire has not unlimited powers of alienation of his concession, and cannot dispose of it as freely as if he were an absolute owner. In some countries, as in Spain for example, the concession is considered to be the absolute property of the concessionaire, who can deal with it and dispose of it as he may think fit, subject only to the payment of dues, which vary with the nature of the mineral conceded and with the area of the concession; when the mineral is worked, however, a further tax is payable upon the amount of mineral extracted.

In strong contrast with the wide rights thus conferred is the system under which concessions are granted in Norway.* These are restricted to Norwegian citizens, save in exceptional circumstances, when a Royal Decree authorising the grant of a concession to a foreigner may be obtained. The concession is granted for a limited period not exceeding 50 years, and at its determination all workings, buildings, and fixtures shall belong to the State ; it is subject to dues levied on a tonnage basis, the amount of which is, however, determined specially in each individual case, and is subject to revision at the end of each period of 10 years. The concessionaire has not the unrestricted right of alienation. It is possible to mortgage a concession, but the rights of the mortgagee are strictly defined, as his power of foreclosure is limited ; it is very doubtful whether a foreign mortgagee could exercise any powers at all. There are many other conditions prescribing that a definite proportion of the directors of a mining company, of the miners, and of the officials shall be Norwegian citizens, and that the majority of the shares of a mining company working a concession shall not be held by foreigners. It is obvious that the value of a concession hedged round with so many restrictions is far less than that of one granted under more liberal conditions.

In the Forest of Dean a system that very closely resembles the concession system prevails. The mineral area of the Forest is divided into definite portions or "gales" ; when any gale is released, any "free miner" —that is to say, any man born within the Hundred of St. Briavels, and who has worked a year and a day in the mines of the district, and is duly registered as a free miner, may apply to the deputy Gaveller for a grant of the gale. Usually the application is made by a

* "Koncessions-loven" (The Law of Concessions), Dec. 14, 1917.

small party of free miners ; the gale is granted to them subject to the payment of a royalty, usually a small one not exceeding 6d. per ton on coal or iron ore, to the Crown ; they then usually sub-let the gale to the individual or company proposing to work it at a royalty which is naturally an increase, often a substantial increase, upon that which they themselves pay. The sub-lessees, as long as they continue to pay the royalty in question, are entitled to hold and work the gale throughout the term of their lease. The position of the gale-holder is clearly defined in the second of the Acts regulating these matters :—*

" The Grant of a Gale of Coal or Iron or of a Stone Quarry shall be deemed to have conferred and shall confer upon the Galee, His Heirs and Assigns, a License to work the mine vein or pit therein comprised, and such Grant shall be deemed to have conferred on the Grantee, His Heirs and Assigns, an Interest of the Nature of Real Estate, such License nevertheless being conditional on the Payment of all Rents, Royalties, and other Dues. . . ." This is exactly the position of a concessionaire when the concession is for an unlimited period, as in Spain, and sets forth the position of such a concessionaire very clearly. It may be noted that, under the above Act, the obligation to pay the rents, royalties, or other dues is a personal obligation upon the person for the time being in actual possession of the gale, whether as owner, lessee, under-lessee, or in any other capacity. The rights of the galee and of those holding a gale under him are thus quite strictly defined, and either can, if necessary, be made the object of a valuation.

* Vic. 1 & 2, 1838, C. 43, and Vic. 24 & 25, 1861, C. 40.

CHAPTER II.

MINERAL DEPOSITS.

Mineral deposits have been exhaustively studied by mining geologists, who have devoted great attention to their modes of origin, and there are in consequence numerous treatises devoted to this subject. The mineral valuer regards mineral deposits chiefly from the point of view of their persistence in depth, the permanence or variability of their dimensions, upon which depends the quantity of their mineral contents, and of the continuance, increase, or decrease in the proportion of the constituents upon which their economic value depends, or, in other words, their richness. It is evident that the value of a vein of gold quartz, for example, depends upon whether the vein persists in depth, whether it continues of a size sufficient to permit of economic working, and whether it contains a sufficient richness in gold to be remunerative. Questions such as these but rarely admit of a definite answer, and the valuer has to content himself with an estimate of the probabilities of a more or less favourable or unfavourable result in respect of each of these particulars; such an estimate can only be based upon experience, and more particularly local experience, because the laws that govern mineral deposition are exceedingly complex, and are affected by numerous conditions which vary in every district; necessarily great assistance is derived from a thorough study of mining geology, which will assist the mining engineer to draw correct inferences from such facts as he is able to ascertain.

In treatises devoted to mineral deposits these are usually classified on genetic principles, but for the purposes of the present work, the simple morphological classification that has long been adopted by miners into: I. Beds, II. Veins, and III. Masses, will probably prove to be the most convenient.

I. BEDS.

Beds or stratified mineral deposits constitute integral portions of the formations in which they occur, being younger than the floor and older than the roof, and are, therefore, subject to the ordinary principles of geological stratigraphy. They are mostly persistent over extensive areas, and, generally speaking, the continuance of a deposit may be inferred with a high degree of probability for as far as the over-lying or under-lying beds are seen to continue with a reasonable measure of uniformity. There are, however, very great differences between different bedded deposits in respect of uniformity in mineral richness, this depending very largely upon genetic conditions. In one class of bedded deposits the value of the deposit has been due to the original nature of the deposit itself. Such is the case, for example, with respect to coal seams; these are, of course, greatly metamorphosed from the original beds of vegetable matter from which they were derived, but the organic constituents to which they owe their present value existed in them from the commencement of their formation. Another example is to be found in the great beds of Minette iron ore, which underlie a very large portion of Lorraine and some of the adjoining parts of Luxemburg. These were probably laid down originally in lakes or shallow seas in the form of a soft hydrated oxide of iron, and although the beds have become consolidated

and undergone physical changes, it is quite possible that they may not have varied very greatly in their composition from the time of their deposition to the present day.

In another class of bedded deposits their value as mineral deposits depends upon chemical changes which they have undergone since their deposition, the change being a metasomatic one—that is to say, one in which the bed as a whole has undergone an alteration in chemical composition, molecule by molecule. A good example of such a deposit is the Cleveland ironstone. This was no doubt originally deposited as a bed of limestone ; subsequently to its formation and to its partial or complete consolidation, it has been subjected to the action of ferruginous solutions which have changed the carbonate of lime by inter-action into carbonate of iron, the change taking place molecule by molecule, and thus leaving the physical structure of the bed undisturbed, so that even the details of the fossils in the beds which were originally calcareous are faithfully reproduced in carbonate of iron. Obviously in such a case it would be quite possible for the bed itself to persist uniformly, whereas the ferruginous solutions may have had access only to a limited area ; as a result the richness in iron of the deposit would decrease as the distance from this zone of access increased, so that the bed would become gradually poorer and poorer in iron, until it became too poor to be worth working as an iron ore. In such cases the valuer has accordingly to consider not only the persistence of the bed itself, but also that of its iron contents. In many cases he is helped by the fact that the decrease in richness takes place with more or less uniformity and regularity, so that careful observation may give some clue to the richness to be expected in any given district. Other examples of this type of de-

posit are the ironstones of Oxfordshire, Lincolnshire, and Northamptonshire. In the last-named case the original limestone bed appears to have been deposited under estuarine conditions, and to have been more or less intermixed with sand ; the proportion of limestone to sand appears to have been a variable one, according to whether the rate of deposition of the sand has been faster or slower than the growth of the calcareous elements, so that all gradations from a sandy limestone to a calcareous sandstone appear to have existed. As the metasomatic change affected essentially the limestone only, variations in richness are apt to be more abrupt and less likely to be subject to uniform gradation than in other cases. It is obvious that facts such as these can only be unravelled by a careful study of the geological conditions.

A third group of bedded deposits consists of beds, the main mass of which is of no economic value, but which contains mineral of economic value disseminated through it. A typical example is to be found in the auriferous Banket beds of the Witwatersrand. These consist essentially of beds of quartz conglomerate which in themselves have no value as a mineral deposit ; the cementing material of the pebbles carries gold and auriferous pyrites in sufficient quantities to be remunerative and hence the entire bed is looked upon as a mineral deposit and is worked as such. In by far the greater majority of instances the valuable mineral was not an original constituent of the bed, but has been subsequently introduced by the action of mineralising solutions. In some cases the values may be distributed with considerable uniformity, but they are much more likely to be irregular, and, generally speaking, to follow no ascertainable law. There are cases known where the mineralising solutions appear to have had access

to the beds along fault fissures, and where the beds are apt to be richer in the immediate neighbourhood of such fissures, but even in these it is scarcely possible to make any statement more precise than the above, and the distribution of the mineral values in the bed does not appear to follow any definite law.

With this class of deposit it appears advisable to group gravel deposits such as auriferous, stanniferous, and gem-bearing gravels. These may vary very widely in character; some are tolerably regular and persist over considerable areas, whereas others are of quite small extent and very patchy. The valuation of the former, like some of the Californian deep leads, admits of a considerable measure of accuracy, whilst in most of the latter instances it is rarely possible to do more than frame rough estimates, which can only be looked upon as general approximations to the order of magnitude of the results to be expected.

II. VEINS.

Mineral veins are the result of the in-filling of fissures in the earth's crust by mineral matter, such filling having been due to hydro-thermal action in some form or other, aided by the effects of a number of causes that tend to deposit mineral matter from its solutions. Fissures that have been filled by the injection of molten matter are known as dykes; it at times happens that the dyke matter may rank as mineral in the sense in which this term is here employed—namely, as indicating the possession of intrinsic value, but such value is always relatively low, unless secondary changes have introduced valuable minerals. For example, whin dykes are quarried for road metal and paving setts, and in this respect the whinstone would rank as a mineral; on the other hand, some of the auriferous deposits of West

Australia appear to be altered and mineralised dykes. The content of a mineral vein or the vein-stuff consists for the greater part of non-metalliferous matter, the gangue, and to a lesser extent of the more valuable metalliferous minerals, for the extraction of which the vein is in fact worked. Metalliferous, or, as they are sometimes called, metallic minerals, are understood to be minerals containing the ordinary heavy metals of commerce, the phrase being used in a commercial and not in a strict chemical sense. Thus spathic iron ore is a compound of carbonic acid with the oxide of the metal iron in precisely the same way as calcspar is a compound of carbonic acid with the oxide of the metal calcium, the word metal in this sentence being used in its chemical sense. To the miner or the metal broker, however, iron is a metal, whilst he does not recognise calcium as such, so that he would call the former a metalliferous mineral and the latter a non-metallic mineral. More precisely it may be said that compounds of bodies which the chemist designates as metals of the alkalies and of the alkaline earths rank as non-metallic minerals, unless they at the same time contain compounds of the so-called heavy metals. In the official lists of mineral production issued by the United States Geological Survey, the term non-metallic minerals is applied to minerals which are not used as sources of metals, even although they may contain metals in the ordinary commercial sense; for example, an ochre used as a paint will be described as a non-metallic mineral, although it is in fact an oxide of iron.

The gangue of a vein invariably contains quartz, and may contain other minerals like calcspar, fluorspar, barytes, mica, felspar, chlorite, etc.; although some of these, like fluorspar and barytes, may have intrinsic value, and the vein is then mined so as to produce them as

far as possible in good marketable condition, they are, nevertheless, classified as gangue. Spathic iron ore presents a certain amount of difficulty in respect of its classification; it is sometimes the principal mineral constituent of a vein—as, *e.g.*, in the Brendon Hills, Somerset—at other times it is merely an accessory, and even a troublesome accessory, as in the lead and zinc veins of Nenthead. Its position in a scheme of valuation is necessarily widely different in the two cases.

The fissures themselves are for the most part nearly or quite vertical, being rarely flatter than 60° to the horizontal. Fissures may occur in stratified or igneous rocks, and may pass indifferently from the one to the other or through a series of different strata. At times, however, fissuring has taken place between two adjoining strata, and in such cases a vein is formed parallel to the planes of stratification; this is spoken of as a bedded vein. Veins vary in width between very wide limits; they frequently extend for miles in length, and generally terminate by narrowing or by impoverishment until they become unworkable rather than by actual disappearance, unless they are cut off by some change in the character of the country rock. In the same way their extension in depth is limited more often by the failure of their payability than by actual disappearance. It is generally found that a vein that extends for a considerable length also holds well in depth. As is but natural from the mode of their origin, mineral veins are by no means regular either in their strike or their dip, and the same vein may undergo very great variations in its width. It is generally necessary to take average strikes, dips, and widths; care must be taken not to confuse the strike of a vein with the direction of its outcrop, as the two only coincide when the vein is vertical or the surface of the ground hori-

zontal; the dip is always measured to the horizontal; the width of a mineral vein is always measured at right angles to the walls.

The distribution of the metalliferous minerals within the vein-stuff itself is very variable; sometimes it occurs more or less evenly disseminated through it, forming what the North-country miner calls a "brangled" vein. Sometimes the ores form ribs or strings, which in some veins are apt to follow the hanging wall, in others the foot wall, in others are near the middle of the vein, and in others again change their position capriciously. Sometimes the ores are concentrated into irregular patches within the vein. In all cases, however, there is a marked tendency towards an alternation of poor streaks and rich streaks or shoots in the length of the vein, these shoots being sometimes at right angles to the strike, but more often pitching at an angle, nearly always a steep angle, along the strike of the vein. The importance of paying attention to these features in the valuation of a mineral vein cannot be over-rated, and the character and disposition of the shoots in a vein must always be the subject of careful study, otherwise most serious errors may easily be committed in such valuations. Sometimes the pay-shoots are long, sometimes quite short, whilst in either case the intervals between them may be very considerable or relatively short; sometimes the intervals are absolutely barren, whilst in other cases there is merely a comparatively slight alternation in value. In examining a vein that has been but little opened up, if the behaviour of parallel veins in the same district and carrying similar vein-stuff is known, it may be accepted as a good guide to indicate what is likely to be the character of the vein under examination. Another most important consideration is the fact that the nature of the vein-stuff

is often profoundly influenced by that of the country rock traversed by the vein, and the mining engineer should always be prepared for the possibility that a change in the country rock may affect very seriously the character and value of the veins traversing it. Extreme cases may be quoted like those of the silver-bearing veins of the Kongsberg district of Norway; these are narrow veins, rarely exceeding 1 foot in width, traversing beds of mica schists, intercalated between which are certain narrow beds rich in iron pyrites. The veins are practically barren, except where they traverse these pyritic beds, and there they are extremely rich in silver ores, particularly in native silver; they have been worked for centuries, and are still productive to-day. It is needless to point out how greatly the value of such a vein might be under- or over-estimated by anyone not familiar with this type of occurrence. Again, the lead-bearing veins of the North of England traverse the so-called "Lead Measures," the Carboniferous Limestone, which here consists of an alternation of beds of limestone, sandstone, and shale. In some of the most important districts—*e.g.*, in Alston and Weardale—the beds carry payable lead ore mainly in the limestones, only rarely in the sandstones, and hardly ever in the shales; this feature is so strongly marked that the most important of the limestone beds, the Great Limestone, has by itself produced more lead than all the other sills of the Lead Measures together. When, as is often the case, the vein is the filling of a fault fissure, the fact that one wall consists of limestone or sandstone whilst the other is of shale is often sufficient to cause the vein in that part to be well mineralised. On the other hand, in some of the districts—*e.g.*, in Teesdale—certain of the sandstones above the Great Limestone are more productive than the Limestone itself. Here

again it is obvious that anyone not familiar with this type of occurrence and with the character of the veins in the district could easily fall into serious miscalculations.

Both as regards horizontal and vertical distribution, the old miners' adage that you usually find "ore against ore" is well worth bearing in mind. It will be sufficiently obvious, both from the mode of occurrence of mineral veins and from the character of their filling that the method of proving mineral deposits by means of vertical or nearly vertical boreholes, so admirably adapted for the investigation of beds, is useless for veins, and that the latter need far more extensive proving than do the former before an equally reliable opinion can be expressed as to their probable value.

III. MASSES.

As their title implies, masses are irregular bodies of mineral, having no regularly defined shape capable of being represented by a simple geometrical figure, but tending in most cases to be more nearly equi-dimensional than either beds or veins, which may be described as bi-dimensional. The great majority of masses owe their origin either to the filling of cavities or to the metasomatism of soluble rocks (or both actions together), or else are genetically connected with igneous rocks, and produced by segregation, pneumatolysis, or hydro-thermal action, or some combination of these. Those of the first-named class are quite irregular in outline, and their dimensions can only be ascertained by actual trial; the best that can be said is that their presence is sometimes, as in the red hæmatite ore bodies of Cumberland, connected with certain definite lines of faulting, and in such cases it is reasonable to presume that the deposits will follow these lines and will not be found to extend very far from them. Deposits of the

second-named class show a somewhat greater tendency to regularity, and are often found to assume a lenticular form. It must not be imagined that this phrase implies anything of the regularity of shape of the lens, but merely that they resemble the lens in this respect that they are widest in the middle and gradually thin out all round; they are perhaps more often boat-shaped than lens-shaped, and are, of course, constantly bent, contorted, broken, and twisted in various ways; nevertheless, such deposits do show sections that are more or less approximately elliptical on three planes, which are frequently approximately at right angles to each other. Other deposits again consist of masses of igneous rock carrying disseminated mineral matter more or less evenly distributed through them; here again the shape of the deposit is that of the igneous mass, or, at any rate, of that portion of it that carries the valuable mineral. As one example may be quoted the diamond-bearing pipes of Kimberley, where the shape is quite well marked, the dimensions readily ascertainable, and the richness of each pipe relatively constant, another of quite opposite character being the auriferous albite-diorite of the Alaska-Treadwell region, the outline of which is entirely undefined. Tinstone quite often occurs in this way disseminated through altered granites.

In dealing with irregular deposits, therefore, it is exceedingly difficult to predicate anything as to their extent or their character, so that any valuation must be based upon the results of actual proof, which in most cases means a considerable number of boreholes, trial shafts, or other workings, or both methods combined. For example, the very large, relatively flat-lying deposits of iron ore in the Mesabi range, Minnesota, have been extensively prospected by means of boreholes in order to determine the limits and estimate the probable tonnage

of the deposits. Similar methods have been used in the irregular masses of zinc blende occurring in limestone in Wisconsin. Again, these methods have been used in the extensive porphyry copper deposits in the Western States ; thus the masses of disseminated copper ore in porphyry in the Ely District, Nevada, estimated at some 95 millions of tons, form irregular, more or less flat-lying bodies, which have been proved by means of several hundred drill holes.* The highly irregular masses of red hæmatite occurring in the Mountain Limestone of Cumberland are, as stated above, known to follow more or less closely several well recognised lines of faulting, but their existence or non-existence has in every case to be determined by boring. Boring in this district is not carried out as systematically as in the American examples just quoted ; a deposit that has been proved by one or two boreholes is usually opened by means of a trial drift from the nearest conveniently situated working.

This brief review of the mode of occurrence of mineral deposits may, therefore, be summarised by saying that it is possible by proper investigation and careful geological studies to draw reasonable inferences in the case of :—

I. Beds,	.	As to their extent and the persistence of their contents.
II. Veins,	.	As to their extent, but not as to their contents.
III. Masses,	.	Neither as to their extent nor as to their contents, except where definitely proved by driftings, borings, or sinkings.

* "The Geology and Ore Deposits of Ely, Nevada," by Arthur C. Spencer. U.S. Geological Survey Professional Paper 96 (1917), p. 114.

SUPERFICIAL ALTERATION OF MINERAL DEPOSITS.

It is well known that the circulation of water within the earth's crust falls into two categories, the superficial and the deep-seated. The superficial is due to water falling on the earth's surface, sinking in, and flowing out again at some lower level. A portion of the water, however, is drawn into the earth by capillary action, becomes heated, and ascends again through fissures or vents, either as heated water or as steam, thus giving rise to a system of deep-seated water circulation. Whereas the former is largely affected by rainfall and seasonal change, the latter is practically independent of these, and maintains a relatively permanent level. This latter water may be hot or warm, usually carries various minerals in solution, and is practically free from oxygen. The former, the atmospheric water, is generally well charged with both oxygen and carbonic acid in solution. Such water is, therefore, capable of exerting a strong oxidising influence upon any oxidisable mineral with which it may come into contact, and can thus bring about important changes in the upper portions of mineral deposits from their outcrops down to permanent water level or even somewhat below it. The extent of such change is affected by the temperature both of the water and of the upper portion of the earth's crust, and by the nature of the country rocks that enclose the mineral deposit, and it may manifest itself in a number of ways, producing in some cases surface impoverishment and in other cases surface enrichment.* When oxidation tends to form freely soluble salts not readily precipitated, as when zinc sulphide is oxidised

* U.S. Geological Survey Bulletin 529, "The Enrichment of Sulphide Ores," and Bulletin 625, "The Enrichment of Ore Deposits," by William Harvey Emmons.

to zinc sulphate, particularly in the presence of an ample flow of water and readily permeable country rock, surface impoverishment results. If conditions exist under which readily soluble salts form, which are capable of reprecipitation at a lower level, especially by the reaction of the as yet unoxidised minerals, surface impoverishment combined with enrichment of the lower level of the deposit will be brought about, particularly if the country rock is relatively impervious to water; many examples of this action are to be found amongst copper deposits. When the products of oxidation form or leave insoluble or difficultly soluble minerals, or form compounds which if themselves soluble, readily form insoluble compounds, surface enrichment ensues, as is notably the case with silver-bearing deposits, and when such oxidation takes place *pari passu* with the degradation of the deposit, the surface portions of the deposit show enrichment, as is often the case with gold ores. The three factors that play the most important part in these phenomena are the composition of the minerals in their unaltered state, or the "protore," as it has been well designated by Mr. F. L. Ransome,* the quantity of atmospheric water traversing the deposit, and the nature of the country rock. The term "protore" is here used more loosely than it is by Mr. Ransome, who restricts it to such portions of the deposit as are too poor to be payable, and are hence not properly called ore; as the possibility whether a given portion of a deposit can or cannot be made to pay must vary from time to time, and as the object here is to call attention to the effect of surface action in altering values rather than to the intrinsic value of the material affected, the term protore is here extended to cover all classes of

* "The Copper Deposits of Ray and Miami, Arizona." U.S. Geological Survey Professional Paper 115, p. 156.

primary ore whether actually payable or not, and is simply used as a convenient term for a primary ore before it has undergone surface change of some kind or other.

Metals occur in nature mainly in three forms, as native metals or alloys, as oxidised compounds, and as sulphides, the last-named often more or less complex. The only occurrences of native metals in protores are those of gold and platinum; the presence of other native metals is due to subsequent reducing reactions, and is confined to the upper portions of the deposits, practically the only exception to this statement being found in the deposits of native copper in the Lake Superior region. In fact, gold and platinum occur but rarely as true chemical compounds; gold sometimes occurs in a condition of most intimate union with iron pyrites not apparently as a compound, but in a state analogous to a solid solution, in which condition it is not capable of amalgamation by mercury. Assuming that a deposit of such a pyritic gold ore outcrops at the surface, and is thus subject to atmospheric water, the following action will take place:—The iron pyrites will be oxidised to ferrous and ferric sulphates, which in turn will form basic sulphates and ultimately ferric hydrate and oxide, the gold being liberated from its previous condition of intimate union, so that the portion of the deposit above permanent water level will consist of the quartzose or other gangue deeply stained red or reddish-brown by the oxides of iron, and containing free amalgamable gold disseminated through it. Concurrently the gold, especially from that part of the deposit that is being weathered away, is dissolved, in all probability, by the ferric sulphate aided by any chlorides that may be present, and is immediately thrown down again by the reducing action of the sulphides

directly below the surface which have not yet been completely oxidised. The result will be a relatively shallow zone of greatly enriched oxidised free-milling ore, followed by a zone of oxidised free-milling ore relatively slightly enriched, which will as a rule extend down to about permanent water level, whilst below this comes the normal pyritic protore, which will probably require cyanidation for the extraction of its gold contents. There are few gold fields that do not afford examples of this surface effect, and the Witwatersrand may be cited as a striking instance. Here certain portions of the outcrop carried as much as 200 ozs. of free gold to the ton, but only through a depth of a few feet; the oxidised zone immediately below the surface extends to varying depths in different parts of the Witwatersrand, in places down to about 300 feet, the average being probably about 100 feet, and its average content about 15 dwts. of gold to the ton, whilst the unoxidised pyritic protore probably averages about 6 dwts. to the ton. In West Australia, on the other hand, where the gold is associated with tellurides in which the gold itself is chemically combined, the oxidised zone below the immediate surface is in many cases one of well-marked impoverishment. Crushings of outcrop ore gave returns of 15 ozs. per ton; immediately below this came oxidised ores carrying from 6 to 19 dwts. per ton, the zone of oxidation ranging from 70 to 250 feet in depth, and below this again the protore was about double as rich as the overlying oxidised region below the rich superficial zone.

The metals that occur as primary oxidised ores fall into two groups, one comprising elements such as tin and titanium, and the other iron and manganese. The former produce exceedingly stable compounds, which are practically unaffected by atmospheric agencies; in

their case there is little or no surface enrichment or impoverishment due to chemical action, though there may often be surface enrichment due to mechanical causes. Thus the degradation of stanniferous granitic rock may well result in the accumulation of tinstone in the immediately underlying portions of the ore-body, so that the upper layers, partially decomposed, of such a deposit are likely to be richer than the normal protore met with in depth.

Deposits containing iron or manganese in the state of maximum oxidation, such as red hæmatite or pyrolusite, are in the same way practically unaffected by atmospheric agencies, but if the original deposits contain the metals in a lower state of oxidation, say as the carbonates, spathic iron ore and rhodonite respectively, these protores will be decomposed and oxidised, and the surface portions of the deposits will consist of the hydrated peroxides. Numerous examples of this form of change are to be found amongst the iron ores. For instance, the famous brown hæmatites of Bilbao are due to the oxidation and hydration of the ferrous carbonates which constitute the bulk of the ore in depth. If finely ground spathic ore is suspended in water and a current of air and carbonic acid is passed through it, the ferrous carbonate will be converted into ferric hydrate; the ferrous carbonate appears to dissolve in the first instance as ferrous bicarbonate, which is subsequently oxidised and precipitated as ferric hydrate. Veins of spathic ore are practically always altered at their outcrops, often down to considerable depths, as, for example, in the Gergal district of Almeria. It will generally be found that the oxidised portion of the vein is considerably wider than the vein proper of spathic iron ore, the vein walls being frequently attacked and oxide of iron deposited in their place. A quite narrow vein of spathic

ore may thus be represented at the outcrop by a wide "blow" of brown hæmatite, which may easily give a totally misleading impression of the vein proper. Impure carbonate ores of iron undergo the same change; thus the Northamptonshire ores were for many years, as long as they were worked open-cast, known only as brown hæmatites, whilst in depth they are found to consist of greenish-grey ferrous carbonate and silicate. This oxidising action may at the same time be attended with either an improvement or occasionally with a deterioration in the quality of the ore. In many cases the percentage of iron in the ore goes up, but the value of the ore depends less upon the actual percentage of iron than upon the ratio of iron to silica; if the percentage of iron in an ore is diminished, say, by an increase in the readily volatile constituents that are driven off in the calcining kilns, this is a matter of comparatively little importance, but if the percentage of iron is diminished by the leaching out of some of it whilst the silica is left unaffected, the value of the ore is thereby lessened, because the ratio of slag-forming material to iron is thus rendered higher, involving additional expense in smelting the ore. For example, in the neighbourhood of Irthlingborough, where the same bed of Northamptonshire iron ore is worked both by open-cast and by underground mining, the oxidised brown hæmatite in the former averages 42·5 per cent. of iron and 12·55 per cent. of silica, whilst the unoxidised green ore worked underground averages 35·64 per cent. of iron and 9·39 per cent. of silica; the ratio of iron to silica in the former is 3·26 : 1, and in the latter 3·79 : 1, so that the ore, which in its raw state is the lower in iron, is really the better ore for the smelter. On the other hand, the ironstones of Lincolnshire and Oxfordshire that are being worked at present are got solely

by open-cast mining, and are in the condition of brown hæmatites, whereas in the deeper lying portions of the deposits the iron occurs as ferrous carbonate; both these ores are highly calcareous, and the oxidation of the iron has been attended by the leaching out of a certain proportion of the lime as a bicarbonate, so that here the oxidised ores are definitely enriched by atmospheric action.

The effect of this surface action is naturally most marked when the original ores occur as sulphides, these being readily oxidised by the atmospheric oxygen dissolved in the water. Most veins, when "mineralised" at all—that is to say, when carrying any metalliferous minerals of economic importance—contain more or less iron pyrites; this is readily oxidised, forming first a sulphate, then apparently a basic sulphate, and finally ferric hydrate or brown hæmatite, which may pass more or less into anhydrous red oxide. The outcrop of such a deposit, carrying iron pyrites in depth, will, therefore, generally appear as a "gozzan," a more or less cellular quartzose mass coloured dark brown or red and impregnated with the peroxidised iron compounds. Sometimes this gozzan also carries oxidised ores of other metals, as, for example, carbonates, phosphates, arseniates, etc., of copper, sulphates, carbonates, phosphates or arseniates of lead, etc., and in this case it affords a sound indication of the ores likely to be met with in depth. The presence or absence of such oxidised ores depends upon many accessory conditions, amongst them being the composition and structure of the country rock, and their absence is by no means evidence that the deposit does not carry such metals in depth. The gozzans of many very important copper deposits show no indications of copper at all. The presence of iron pyrites in depth is, however, so constant an indication

of mineralisation and of the probable presence of sulphuretted ores of other metals, that miners have long learnt to associate the presence of a highly ferruginous gozzan with a rich ore deposit underlying it, and the fact has passed into a miner's proverb in many countries.

Copper deposits in many districts afford characteristic examples of surface alteration. The great deposits of cupriferous pyrites in the Huelva region are overlain by a gozzan consisting of an impure brown hæmatite carrying only traces of copper; immediately overlying the unoxidised pyrites is a thin dark pulverulent layer consisting of rich copper ores with a notable proportion of gold and silver. The protore contains an average of 1·5 per cent. of copper and very small amounts of silver and gold; the dark layer is usually between 4 and 8 inches in thickness, consisting chiefly of chalcocite, and in one example carried between 10 and 20 dwts. of gold and about 40 ozs. of silver to the ton. Secondary reactions have played an important part in the so-called porphyry deposits of Arizona,* Nevada,† and Utah. At Ely, Nevada, the protore is a monzonitic porphyry carrying disseminated copper minerals, chiefly chalcopyrite, in quantities sufficient to give 0·5 per cent. of copper in the unaltered rock. The superficial portions have been leached to depths ranging from 20 to 200 feet, until they average only 0·2 per cent. of copper, and the copper has been redeposited below the leached zone, forming an enriched zone carrying chalcocite and chalcopyrite, about 300 feet thick and averaging about 1·5 per cent. of copper.

A good example of the effect of surface enrichment

* U.S.A. Geological Survey Professional Paper 115, "The Copper Deposits of Ray and Miami, Arizona," by F. L. Ransome.

† U.S.A. Geological Survey Professional Paper 96, "The Geology and Ore Deposits of Ely, Nevada," by A. C. Spencer.

is afforded by the Broken Hill Mines of New South Wales. Here the protore contains 5 to 7 per cent. of lead, 14 to 30 per cent. of zinc, and an average of 20 ozs. of silver to the ton. The upper portions of the sulphuretted ore-body, on the other hand, carried about one-and-a-half times as much lead as zinc. The gozzan consisted of oxidised ferruginous and manganiferous ore with about 18 per cent. of lead and from 7 to 45 ozs. of silver to the ton, going up in places to 80 ozs., and no zinc. The chemistry of the changes in the upper portion of these deposits will be readily intelligible from what has been said above.

An interesting study of the surface enrichment of silver ores in Colorado has recently been issued by Edson S. Bastin,* in which attention is drawn to the criteria by which primary and secondary silver ores may be distinguished from each other.

These examples might be multiplied almost indefinitely, but enough has been said to show how important is the effect of atmospheric action upon the value of an ore-body, and to emphasise the need of studying a mineral deposit below permanent water level before attempting to assign an average value to its mineral contents.

It is not ores alone that are affected by surface action; practically all mineral deposits are liable to be affected thereby in some way. Thus the diamond-bearing pipes of the Kimberley district consist in depth of the so-called blue ground, a relatively hard, dark blue altered igneous rock, some of the mineral constituents of which contain ferrous silicate. Near the surface these have been decomposed, and the ferrous iron altered to ferric, producing a yellowish-white, soft

* "Silver Enrichment in the San Juan Mountains, Colorado." U.S. Geological Survey Bulletin 735 D (1922).

mass, the so-called yellow ground, which usually extended to a depth of 50 or 60 feet. Both the yellow and the blue ground contain disseminated diamonds, but the former is by far the more easily and cheaply worked of the two, and is, therefore, capable of yielding a considerably greater profit for the same contents of diamonds.

Many coal seams are profoundly altered at their outcrops, generally very much for the worse, a good seam of coal being often represented at its outcrop by a thin soft streak of carbonaceous shale, so that its true character is entirely masked; this alteration may at times extend to quite considerable depths, and the change from the worthless surface streak to the normal coal seam is often a very gradual one.

FAULTS.

When that portion of the earth's crust, in which a mineral deposit occurs, is faulted, the mineral deposit is naturally dislocated in the same way as the country rock by the faults that may happen to traverse it, and these may affect the value of the deposit in various ways. Bedded deposits (including also veins that lie very flat and in this respect resemble beds) are dislocated vertically, so that a certain amount, and at times a very considerable amount, of dead work in the form of inclined drifts or shafts is rendered necessary, in order to enable mining operations to be continued. Even small faults or hitches may add somewhat to the cost of working the deposit, and in this respect a greatly faulted region is less valuable than one not so affected, because the working costs are thus increased to a greater or less degree. The quantity of mineral available in a given area is also affected, as a rule not to any very serious extent, although a fault may readily produce an

alteration in the total area of the mineral bed underlying a given area of the surface. A normal fault or slide shifts the two portions of the bed away from each other, and this gives rise to a barren strip of ground, the width of the strip depending upon the throw of the fault, its hade or inclination to the vertical, and its own actual width. In Fig. 1, if v be the throw of the fault, D C, measured vertically, d its true width, E B, measured

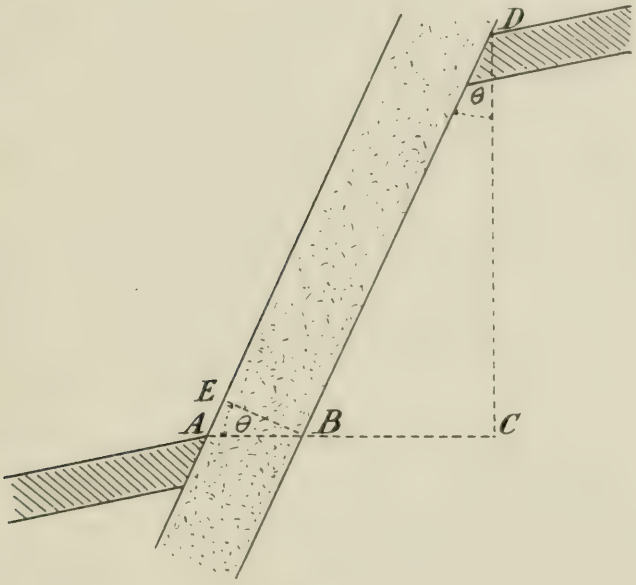


Fig. 1.

at right angles to the fault plane, and θ its hade, B D C, in degrees, measured to the vertical, the width of the barren strip, A C, is $v \tan \theta + d \sec \theta$; it is, therefore, equal to d when the fault is vertical or $\theta = 0$, and increases as the fault flattens. It will be understood that in the diagram the width of the fault, d, is represented as very considerable for the sake of clearness; it may be practically zero, and rarely exceeds 10 feet or so,

although greater widths are known to occur. The exception to the rule that a normal fault produces a barren

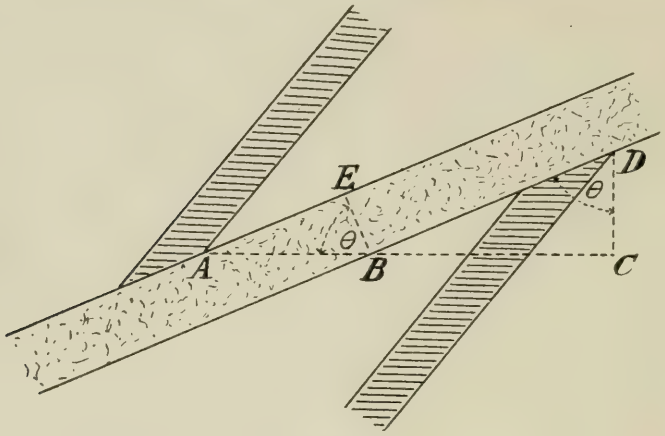


Fig. 2.

strip is found in the unusual case, when the bed dips more steeply than the fault; when this occurs, as shown in Fig. 2, the deposit is doubled over a width,

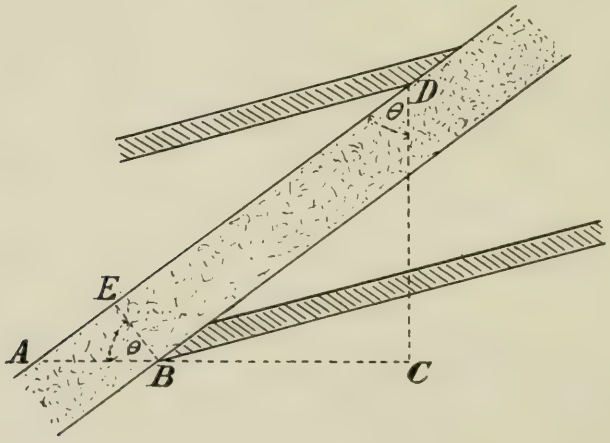


Fig. 3.

A C, equal to $v \tan \theta + d \sec \theta$. A reversed fault or over-thrust in the same way, as shown in Fig. 3, pro-

duces a doubling of the bed over a width, A C, equal to $v \tan \theta - d \sec \theta$. These calculations are only strictly true if the movement is purely along the fault plane; if the movement is diagonal, or if there is in addition a lateral or torsional motion, the calculation will necessarily be affected thereby; generally speaking, however, the above methods will give results quite as nearly accurate as is necessary for the purpose. The effect of an ordinary fault on a vein (or very steeply dipping bed) is noticeable more especially in plan; the direction of motion caused by the fault may be resolved horizontally and vertically. The vertical component shifts the horizontal portions of the vein up or down relatively to each other, but as the main body of the vein lies in that same direction, the effect upon the vein as a whole is not noticeable, whereas the horizontal component shifts the vein in plan, and in this way introduces the need for additional dead work. As a rule, there is little or no barren interval produced along the direction of strike; there may be such a gap, or the vein may be doubled over a certain distance according to the relative directions of strike and dip of the planes of the vein and the fault respectively, but this effect is practically speaking negligible in the great majority of cases. The principal effect of a fault is to introduce a certain amount of dead work in the form of drivages necessary to recover the dislocated portion of the vein and to connect the workings, and the additional expense thereby entailed. Where veins are mineralised only when they traverse certain beds, the effect of faults in displacing these beds, and thus indirectly affecting the richness of the veins, must not be forgotten.

In the case of irregular deposits, the effect of a fault cannot be stated in general terms. Each individual case must be studied by itself, and in complex cases a con-

siderable amount of work may be required before the precise effect of the fault upon the deposit can be unravelled. In many cases it will be found that faulting has practically no effect upon the value of the deposit, and accordingly it comparatively rarely enters as an element in the calculation of the value of an irregular deposit.

CHAPTER III.

DATA FOR VALUATION.

Amongst the various data upon which a mineral valuation must be based, perhaps the most important is the quantity of mineral contained within the area or the deposit that forms the subject of the valuation, and the first step in the operation of valuing is generally the preparation of an estimate of quantity. It cannot be too strongly emphasised that it is never possible to determine this figure accurately. This statement does not refer to the fact that dimensions and weights are incommensurable, and can, therefore, never be determined with absolute accuracy; the accuracy here spoken of is of a relatively much rougher kind, being merely that called for in ordinary commercial work. Even within these limits, it is quite impossible to estimate correctly the quantity of mineral that a given property will yield, for this is a figure which is never known, and never can be known, until the whole of the mineral has been worked out from the property in question, when the information is naturally worthless. The mineral valuer has, however, to prepare his estimate of quantity in advance, often before any of the mineral has been worked at all. The figure which he arrives at never does and never can represent the true quantity of the mineral present; it should represent the order of magnitude of that quantity, and is, strictly speaking, intended to be the most probable value of that amount—

that is to say, the quantity which is as likely to be exceeded as to be fallen short of when the mineral has all been worked out. If a mining engineer states that he estimates that a given property contains, say, 1,000,000 tons of mineral, he does not mean to assert that he believes that it contains accurately 1,000,000 tons, but that from the data before him he calculates that 1,000,000 tons is the most probable tonnage that can be assigned to the body of mineral in question. It can be at best only an estimate of probabilities, and never anything approaching to an ascertained fact, and the statement must be taken as governed by the laws of probability. Such an estimate may fairly be compared to the statement that if a coin be tossed one thousand times, the most probable value that can be assigned to the number of heads that is to be expected is five hundred. This does not mean that it is probable that the number will be five hundred, but it does mean that five hundred, though it is improbable, is somewhat less improbable than any other number that can be named; it is, in fact, about 40 to 1 against the number being five hundred, but the odds against it being this number are less than against any other number that can be named, and these odds increase in proportion as the number selected is further removed from five hundred; it is, in fact, an even chance that the number of heads shall be between 489 and 511. The statement that a given deposit is estimated to contain 1,000,000 tons must be taken as implying no greater certainty than is involved in the statement that heads are expected to come up five hundred times in one thousand tosses.

The dominating principle is that which J. M. Keynes * calls the Principle of Indifference and which he defines

* "A Treatise of Probability," p. 42.

in the following sentence:—"The principle of indifference asserts that if there is no *known* reason for predicating of our subject one rather than another of several alternatives, then relative to such knowledge the assertions of each of these alternatives have an *equal* probability; thus *equal* probabilities must be assigned to each of several arguments, if there is an absence of positive grounds for assigning *unequal* ones." The definition is subject to the qualification that "there must be no *relevant* evidence relating to one alternative, unless there is *corresponding* evidence relating to the other." It follows from this principle that an unknown magnitude determined by two observations, each of which is as likely to be accurate as the other, should be equally remote from either; so that if a and b be two observations of a magnitude, the most probable value m of that magnitude is obtained when m is equi-distant from a and b, or, if the difference between a and m and b and m (irrespective of sign) be r and r_1 respectively, when $r = r_1$, or when m is the arithmetical mean between a and b. It can easily be seen that this principle, which may be called the philosophical principle of probability, is in agreement with Gauss's mathematical theory of probability, which states that the most probable value of an unknown magnitude is that value for which the squares of the differences between it and the observed magnitudes are a minimum. According to this principle

$$\frac{d\,(r^2 + r_1^2)}{d\,m} = 2\,(r + r_1) = 0, \quad \text{or} \quad r = -r_1, \quad \text{as before.}$$

The acceptance of the mathematical principle of probability, as governing the estimate of the quantity of mineral to be expected from a given property or a given deposit, enables the calculations of this quantity to be placed upon a reasonable mathematical basis, though

it must in fairness be admitted that it is by no means free from difficulties ; the mathematical theory of probability assumes that a very large (strictly speaking an infinite) number of observations is made to determine a given magnitude, whereas in mining work the number of observations is always very limited, and, therefore, the probability of error in the application of the theory is greater than might at first sight appear. Furthermore, there are not as a rule a number of observations made to determine one unknown magnitude, but a number of separate observations are made of varying magnitudes, from which a true average value has to be deduced. Subject to this proviso, however, the theory of probability and the law of errors can be applied with much advantage.

Thus, let such a purely hypothetical case as the following be assumed :—In order to test the vertical thickness of a coal seam over a length of 1,000 yards, ten boreholes are put down 100 yards apart. The coal measures may be assumed to be overlain by boulder clay, so that no information is available except that obtained from the boreholes. These show that the strata lie regularly, and do not indicate the existence of any faults or other interruptions ; the figures obtained are as follows (these being merely assumptions made simply in order to illustrate the principle) :—

Borehole No.	1	2	3	4	5	6	7	8	9	10	11
Thickness in Inches.	46	42	48	43	47	40	41	45	49	50	44

It will be readily admitted by anyone experienced in proving mineral properties that such a series as the above would be an abnormally large number of bore-

holes to put down, far more than are used in practice, where the expense of boring has to be considered. Yet even along this well-proved line all that is actually known is the thickness of the coal seam at the eleven points in question ; nothing further is known positively about the thickness of the seam between the boreholes. From general knowledge of coal seams it may fairly be inferred that the seam is likely to have a mean thickness not greater than the largest and not less than the smallest of these figures. It is not likely, for example, that the seam will thicken out to 100 inches between the boreholes. On the other hand, it must be admitted that nips, rock-faults and wash-outs may reduce its thickness even to zero between the boreholes, though, again, experience has shown that such abrupt thinning down as this would be, is exceptional, and furthermore, that thinning of a coal seam is often compensated by an increase in thickness on either side. The boreholes have given us the information that the strata run with reasonable regularity, and we may, therefore, infer that the coal seam is likely upon the whole to do the same. Upon the basis of this information it is required to determine what is the most probable average thickness of the seam along the full length of 1,000 yards. The first and last of the thicknesses cut in these boreholes apply only to the thickness of the seam on one side of them, whereas each of the others applies to that of the seam on either side ; only one-half of the " weight " should be attached to these two measurements that attaches to the others, so that the series may be looked upon as one of ten measurements, nine of them being the thicknesses given by boreholes 2 to 10, and the tenth being the mean of Nos. 1 and 11, or 45 inches. According to the principles of probability, the most probable average thickness of the seam will be taken as the mean

of these ten figures, or

$$\frac{45+42+48+43+47+40+41+45+49+50}{10}=\frac{450}{10}=45 \text{ ins.}$$

It is, however, probable that this figure is not really the true average, and that its adoption involves some error. It is, therefore, necessary to determine what this error, usually known as the "probable error," * may be. The definition of this error is as follows :—

The probable error e of an average of a number of measurements is taken to be such that the probability that the calculated average differs from the unknown true average by an amount numerically less than e is equal to the probability that it differs from it by an amount numerically greater than e, that is to say, that the probability that the error lies between $+e$ and $-e$ is one-half. According to a generally accepted formula, the value of this error is given by the expression

$$0{\cdot}6745\sqrt{\frac{\Sigma r^2}{n(n-1)}},$$

where Σr^2 is the sum of the squares of the "residuals," these being the differences obtained by subtracting the average from each of the observations, and n is the number of observations. There is an approximate expression for the value of this error known as Peters' formula, which is

$$0{\cdot}8453\ \frac{\Sigma r}{n\sqrt{n-1}};$$

the latter is often employed when the number of observations is very large, and the calculations of the sum of the squares of the residuals is correspondingly laborious ;

* See *e.g.*, "The Theory of Errors and Least Squares," by Professor L. D. Weld.

this is so rarely the case in mining work that the former expression may generally be applied. The working out of the data above given would be as follows :—

Thicknesses.	Differences from Average.	Squares of Differences.
45	0	0
42	− 3	9
48	+ 3	9
43	− 2	4
47	+ 2	4
40	− 5	25
41	− 4	16
45	0	0
49	+ 4	16
50	+ 5	25
450		108

Disregarding the fact that there were in reality eleven observations and calculating for the sake of simplicity as if only ten such measurements had really been made, the probable error would be approximately

$$0{\cdot}6745 \sqrt{\frac{108}{10 \times 9}} = 0{\cdot}745.$$

The average thickness would, therefore, be returned as $45 \pm 0{\cdot}745$ inches. It is frequently convenient to return the probable error as a percentage of the average, in this case 1·65 per cent. If the calculation had been performed with due regard to the fact that eleven observations were actually made, of which the first and last had only half the weight assigned to them that is given to the nine others, a more correct probable error of $\pm 0{\cdot}704$ would have been obtained. In the present case the difference is negligible.

Obviously the magnitude of the probable error gives

a good indication of the degree of confidence that attaches to the average of the observations. It need hardly be said that with fewer observations and more widely divergent ones the probable error becomes much greater ; thus if only two measurements were available—*e.g.*, 42 and 48 inches—the probable average thickness calculated as above would be 45 ± 2 inches, assuming that the method can be applied to two observations only. The addition or subtraction of the probable error to or from the mean gives an upper and a lower limiting value, and in view of the risks that undoubtedly attach to mining operations, the lower value will in most cases be the one to be adopted, because experience has taught that there are more causes operative to diminish the tonnage of a deposit within a given area than to increase it above an average determined from *a priori* calculations. This consideration may be held to afford some justification for the practice of many mining engineers, who, after calculating average values as closely as the available data permit, are in the habit of deducting a certain percentage "for safety." It has been well pointed out that if the work has been done with the utmost care possible, it is not admissible to deduct anything from the mean result arrived at, and that it would be just as reasonable to add an arbitrary percentage as to deduct it. The determination of the probable error, however, at once removes such deductions from the region of arbitrary allowances. A valuer who states his estimates plus or minus the probable error gives data upon which vendor or purchaser can logically base his calculations of value. The valuer does not necessarily, nor indeed usually, state the probable error in this way in returning the results of his valuation ; he should use it for his own guidance, and it is particularly useful whenever he is called upon to

advise within what limits it is safe or prudent to vary his average figures.

A curious method of determining the thickness of coal seams is given in the United States* Regulations for the Classification and Valuation of Coal Lands, approved by the Secretary for the Interior on February 20th, 1913, this being apparently intended to be a crude method of "keeping on the safe side" in averaging thicknesses. It is stated as follows :—

"Provided that where the thickness of any bed (*i.e.*, of coal) varies irregularly its computed thickness (C T) over any area shall be equal to the average of the measurements (A M) less the sum of the differences between each measurement and the average of the measurements (S D) divided by the sum of the measurements (S) :—

$$C\,T = A\,M - \frac{S\,D}{S}.$$"

The language is as crude as the method, and it is obvious that the sum of the differences in the above sentences must mean the sum of the differences irrespective of sign. This appears from the text of the Bulletin where the method is illustrated by the following example :—

"if the measurements on a given bed in a certain area are 4 feet, 5 feet, 7 feet, 3 feet, 4 feet, 8 feet, 5 feet, 2 feet, 4 feet, 2 feet, 6 feet, and 10 feet, the average of all these measurements is 5 feet ; the sum of the measurements is 60 feet ; the difference between the several measurements and the average 5 feet is 1 foot, 0, 2 feet, 2 feet, 1 foot, 3 feet, 0, 3 feet, 1 foot, 3 feet, 1 foot, 5 feet, which added together give 21 feet (*sic !*) ; so 21 feet divided by 60 has been called the 'modulus of irregularity,' and has been adopted as a factor of

* U.S. Geol. Survey Bulletin 537, "The Classification of the Public Lands," by George Otis Smith and Others (1913), p. 97.

safety; $1 - \frac{21}{60}$ or 100 per cent. − 35 per cent. = 65 per cent.; 65 per cent. of 5 feet = $3\frac{1}{4}$ feet;* therefore $3\frac{1}{4}$ feet is taken as a safe average thickness on which to sell the coal, in other words the thickness which it is highly probable will be reached or exceeded by the coal under any of the land within the area of the measurements given."

It has already been shown that the correct method of dealing with such a series of measurements as the above is to determine the probable error, which in this case, if all the measurements are equally reliable, amounts to 0·47 foot, so that the proper inference to be drawn from such a series of measurements is that the true thickness is $5 \pm 0{\cdot}47$ foot, and it is conceivable that a purchaser would demand that the lower figure thus arrived at should be taken as the basis of a valuation, but there appears to be no reason at all for the purely empirical method adopted by the U.S. Geological Survey.

The method above given for determining the average thickness has been applied to the simplest case, where all the boreholes are equi-distant. When distances and dimensions do not vary very greatly as between each other, this method may probably be applied with safety. If, however, the thicknesses are very divergent the distances between the boreholes should preferably be taken into account. It is usually considered that the cross-section of a seam between each pair of boreholes may be looked upon as a trapezoid, the two parallel sides being represented by the vertical depths, the horizontal distance between which is known. If a series of boreholes has been put down, the thickness proved by which is respectively *a*, *b*, *c*, *d*, *e*, *f*, *g*, etc., units,

* If the clerical error here made is corrected, this figure should read 3 feet 2 inches.

and the distance between the boreholes is respectively m, n, o, p, q, r, etc., units, the average thickness is given by the expression :—

$$\frac{\left(\frac{a+b}{2}\right)m+\left(\frac{b+c}{2}\right)n+\left(\frac{c+d}{2}\right)o+\left(\frac{d+e}{2}\right)p+\left(\frac{e+f}{2}\right)q+\left(\frac{f+g}{2}\right)r+\text{etc.}}{m+n+o+p+q+r+\text{etc.}}$$

This is equivalent to looking upon the section of the seam as being made up of a series of rectangles, the height of each of which is equal to the thickness indicated by each borehole, and the length of which is equal to half the sum of the distance between each borehole and the adjoining one. Under this conception the numerator of the fraction would become :

$$a\frac{m}{2}+b\left(\frac{m+n}{2}\right)+c\left(\frac{n+o}{2}\right)+d\left(\frac{o+p}{2}\right)+e\left(\frac{p+q}{2}\right)$$
$$+f\left(\frac{q+r}{2}\right)+g\frac{r}{2}+\text{etc.},$$

which is evidently equal to the previous expression. Each of the above fractions divided by the total sum of the lengths may be looked upon as "weight" attaching to the respective observations. Thus, if $m + n + o + p + q + r + \text{etc.} = s$, the weighted thickness for the first borehole would be $a\frac{m}{2s}$, that of the second borehole $b\left(\frac{m+n}{2s}\right)$, etc., and the sum of these weighted thicknesses would give the average thickness, taking account of the distances between the boreholes. The probable error attaching to weighted observations is given by the expression—

$$0{\cdot}6745\sqrt{\frac{\Sigma\,(w\,r^2)}{(n-1)\,\Sigma\,w}},$$

where Σw represents the sum of the weights. For example, if the above series of boreholes, instead of being

at uniform distances, had been put down at the distances apart shown by the following table :—

Borehole	1 to	2, ...	20 yards.
,,	2 ,,	3, ...	70 ,,
,,	3 ,,	4, ...	10 ,,
,,	4 ,,	5, ...	40 ,,
,,	5 ,,	6, ...	200 ,,
,,	6 ,,	7, ...	300 ,,
,,	7 ,,	8, ...	189 ,,
,,	8 ,,	9, ...	60 ,,
,,	9 ,,	10, ...	100 ,,
,,	10 ,,	11, ...	20 ,,

The weight attaching to the first measurement of 46 inches is $\frac{20}{2{,}000} = 0{\cdot}01$,

to the second $\frac{20 + 70}{2{,}000} = 0{\cdot}045$,

to the third $\frac{70 + 10}{2{,}000} = 0{\cdot}040$, etc.

The thicknesses measured, the weights attaching to each and the product of the two is shown by the following table :—

Borehole.	Thicknesses.	Weights.	Products.
1	46	0·010	0·460
2	42	0·045	1·890
3	48	0·040	1·920
4	43	0·025	1·075
5	47	0·120	5·640
6	40	0·250	10·000
7	41	0·240	9·840
8	45	0·120	5·400
9	49	0·080	3·920
10	50	0·060	3·000
11	44	0·010	0·440
		1·000	43·585

It is thus seen that the average thickness, allowing for distances between the boreholes, would be 43·585 inches. The probable error is obtained as follows :—

Borehole.	Weights (w).	Differences (r).	Squares of Differences (r^2).	Products ($w r^2$).
1	0·010	+ 2·415	5·832	0·05832
2	0·045	− 1·585	2·427	0·10922
3	0·040	+ 4·415	19·493	0·77972
4	0·025	− 0·585	0·312	0·00780
5	0·120	+ 3·415	11·662	1·39944
6	0·250	− 3·585	12·852	3·21300
7	0·240	− 2·585	6·682	1·60368
8	0·120	+ 1·415	2·002	0·24024
9	0·080	+ 5·415	29·323	2·34584
10	0·060	+ 6·415	41·153	2·46918
11	0·010	+ 0·415	0·172	0·01720
	1·000			12·24364

The probable error is, therefore,

$$0{\cdot}6745 \sqrt{\frac{12{\cdot}24364}{10 \times 1{\cdot}0}} = 0{\cdot}7522,$$

and the average thickness of the seam should, therefore, be written as 43·585 ± 0·752 inch, showing a probable error of 1·72 per cent. Evidently in a case like the above, where the first decimal is in doubt, it would be absurd to carry the calculation to three decimals. It is also clear that it is unnecessary to apply such refinements as here indicated unless the number and accuracy of the measurements warrant their use.

The laws of probability may also be applied in those difficult cases where, among a series of comparatively concordant observations, there are one or two very discordant ones. Thus, if it be assumed that a set of boreholes has been put down and has given results as shown on p. 46, except that borehole No. 10 has given

a thickness of 100 inches instead of 50 inches, the question then presents itself how this one very discordant measurement should be dealt with. It is often possible that such a figure may be the result of a mistake, and if there is any question as to its correctness it would no doubt be rejected. It is, however, in most cases possible to make practically certain that the discordant value was actually obtained, and that it cannot be due to an error of observation. For instance, if all the boreholes had given satisfactory cores and these cores had been carefully preserved, it would be possible to check the measurements. Some engineers would always include the divergent measurement on the grounds that if the thickness of 100 inches has been found in one place, it is quite likely that it or some similar thickness will occur in other parts of the bed, and that its inclusion is necessary for getting a correct average. Others again would exclude it on the grounds that the close concordance of the other values shows that it must be quite abnormal and that no such figure is likely to occur elsewhere, so that a more correct average would be obtained by its rejection. In many cases geological knowledge of the nature of the deposit or of its mode of origin may be most helpful, as it may indicate either that the deposit is quite likely to be liable to considerable variations in thickness, or that it is highly unlikely that such variations would characterise the deposit. Such considerations will often determine whether a divergent figure should be included or excluded; occasionally, however, no such evidence is available, and the decision must be arrived at solely on the numerical data; in such cases the laws of probability will render assistance. Thus, if the measurement found in borehole 10 is included, the average thickness comes out at 49·55 inches, and the probable

error of that average at 3·44 inches ; if borehole 10 be disregarded, and the average of the other ten boreholes be taken as the true average, this works out at 44·5 inches, and the probable error at 0·65 inch. Some criteria based on the increase of the probable error by rejection of an observation have been suggested, but it would appear that the most satisfactory method is based on the probable error of a single observation in a series of observations, this probable error being equal to $\sqrt{n}$ times the probable error of the average of all the observations when there are n observations. The principle to be applied is well described by Mr. David Brunt * as follows :—" The probability of an error greater than $5\,r$, where r is the probable error of a single observation, is ·001, and the probability of an error greater than $3{\cdot}5\,r$ is ·018. Thus, only one observation in 1,000 should have an error as great as $5\,r$, while one error in 55 should have an error as great as $3{\cdot}5\,r$. These numbers form the theoretical basis of the following rule, which is advocated for general use by Wright and Hayford (Adjustment of Observations) :—' Reject each observation for which the residual exceeds 5 times the probable error of a single observation. Examine carefully each observation for which the residual exceeds 3·5 times the probable error, and reject it if any of the accompanying conditions are such as to produce lack of evidence.' This criterion has the merits of simplicity, ease of application, and a fairly sound theoretical basis. For a moderate number of observations it cannot be said to be too sweeping."

The probable error of a single one of the above series of eleven observations is $3{\cdot}44\sqrt{11} = 11{\cdot}41$. The residual for the doubtful observation is $100 - 49{\cdot}55$ or 50·45, which thus comes very near to the limit as given

* " The Combination of Observations," by David Brunt, 1917.

above. The general evidence may be held, therefore, to tend to the rejection rather than the retention of the observation, but on purely numerical grounds there is no very strong argument either way. Probably most valuers might be inclined to reject the divergent figure and to accept the lower average thickness as the more reliable, having regard to the fact that the error, if any, would be on the side of safety.

Necessarily, the above principle must always be applied with caution, and must not be strained so as to override the dictates of commonsense; it is, moreover, only properly applicable when the total number of observations after the rejection of the divergent ones is still sufficiently large to enable their average to be used with some degree of confidence.

The principles of probability as above enunciated must always be taken as the basis for estimating the probable quantity of mineral that a given property may yield; it will, however, have to be applied very differently according as one or other of the three main types of deposits is the subject of the calculations, and each of these must accordingly be considered separately.

BEDS.

In this case the superficial area to which the calculation is to apply is always accurately known or can be accurately ascertained by an ordinary survey if the subject of the valuation is a definite royalty or concession; if it applies to an entire mineral field or deposit the area can even in that case be determined with a very considerable degree of accuracy, so that the main doubtful element in determining the volume is the thickness of the deposit. It is, of course, always the area as projected on a horizontal plane which is here

being considered ; strictly speaking, in dealing with very large mineral fields, the curvature of the earth's surface ought to be taken into account, but the data available are never so accurate as to warrant such refinement in calculation. If a given superficial area is underlain by a horizontal bed, the probable average thickness of which has been determined, the area of the bed is equal to that of the surface ; the bed is looked upon as a thick sheet of known area having a thickness equal to the probable average thickness, and the volume of the bed is, therefore, obtained by multiplying the area by

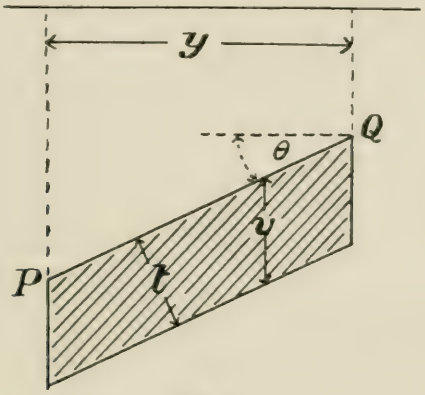


Fig. 4.

the thickness. If the bed is inclined, the area thereof included within a given superficial area is proportionately greater ; let the area of the given surface be A square units (the foot is usually adopted for this purpose in this country and in America, and the meter on the Continent). It is always possible to determine a rectangle of length x parallel to the strike of the bed, and breadth y parallel to the direction of dip, so that $x\,y$ is equal to A ; if the dip of the bed is θ degrees to the horizontal, the actual breadth of the bed, P Q (Fig. 4), becomes $y \sec \theta$,

and the area of the bed $x\,y$ sec θ. If t be the true thickness of the bed, its volume is $t\,x\,y$ sec θ or A t sec θ. As a rule, however, it is much easier to determine the vertical thickness of the bed (v) than its true thickness, the former being given by boreholes, cuts, shafts, etc.; since $v = t$ sec θ, the volume of the bed is given by the expression $v\,x\,y = v$ A, and, to employ this, it is only necessary to determine the superficial area of the property or deposit and the vertical thickness of the bed, so that a determination of the dip is quite unnecessary. In the very numerous cases that occur in which the dip varies from place to place, the determination of the average dip is by no means an easy matter, whilst, as already shown, the average vertical thickness can generally be determined. This is, therefore, by far the simplest method of calculating the volume of a bedded deposit. As has been stated, the surface area of a property to be valued can, as a rule, be readily ascertained; it is, however, not always the case that the whole of the area in question is underlain by the bed of mineral. Occasionally the bed outcrops within the property so that a portion of the surface is definitely not mineral-bearing; this may be due either to the bed being inclined, or, if the bed is horizontal, it may outcrop along the inclined surface of the property, *e.g.*, along the sides of a river valley. In such cases the best plan is to lay down the outcrop accurately by survey (a plane table survey may be recommended for this purpose), and to determine the area underlain by the mineral bed by means of the planimeter. If the bed is a very thick one and the slopes are gentle, the outcrop will include a considerable quantity of the bed wedged-shaped in cross-section between the outcrops of the floor and the roof of the bed, as at A B C in the diagrams in Figs. 5 and 6, the first of which shows

a section of an inclined bed outcropping on horizontal ground and the second that of a horizontal bed outcropping in side-long sloping ground. The area

$$A\,B\,C = \frac{t^2 \cot \theta}{2},$$

and can thus be calculated, the full thickness of the bed putting on beyond A; the expression applies to all cases generally, if θ be the angle between the plane of the surface and that of the bed; if the true thickness t be unknown, it can be calculated from the

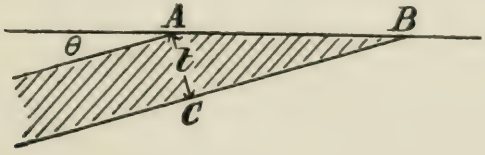


Fig. 5.

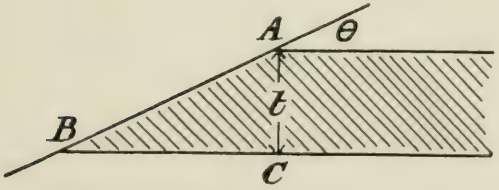


Fig. 6.

width of the outcrop A B and the dip of the bed. Generally speaking, it will be near enough if the outcrop line be plotted midway between such points as A and B, and if this line be taken as the actual edge of the outcrop in measuring up the area. It sometimes happens that the outcrop of the mineral is blind, as, for example, where the upturned edge of an inclined coal seam lying between similarly inclined beds of coal measures is covered by a blanket of boulder clay. In this case the probable position of the outcrop must be calculated

from known geological data obtained from boreholes or from the known depth of the seam at various points and its dip at these points, or, failing these, its average dip. The assumed line of the blind outcrop thus arrived at must be plotted and dealt with as above. Known faults, nips or other "wants," must be calculated and duly allowed for.

It is sometimes necessary to leave certain areas of mineral unworked for various reasons. Occasionally this is prescribed by the conditions of tenure of the deposit; thus in a mineral lease it may be provided that a certain width of mineral shall be left as a barrier against adjoining royalties, or a landowner may stipulate that no mineral shall be wrought within a certain distance of his residence. Sometimes it is necessary to leave blocks of minerals unworked to support roads, railways, canals, reservoirs, etc. The necessity for doing this is due to the statutory obligation on the mine worker not to damage works of public utility. The legal rights of the various parties are laid down in a series of Acts—namely, the so-called Mining Code of the Railway Clauses Consolidation Act of 1845,* Ss. 74 to 85, the Water Works Clauses Consolidation Act, 1847,† Ss. 18 to 27, and the Public Health (Support of Sewers) Amendment Act, 1883.‡ Broadly speaking, the effect of these Acts is that when land is purchased for the construction of a railway line or any similar purpose, minerals do not pass with the land unless specifically conveyed; the owner of the minerals or his lessee can work up to a prescribed distance, generally 40 yards from the railway or works, but before advancing beyond such prescribed limit he must give notice to the railway or other company of his intention so to do, when the

* 8 & 9 Vict., C. 20. † 10 & 11 Vict., C. 17.
‡ 46 & 47 Vict., C. 37.

company has the option of either allowing him to proceed and takes upon itself the risk of damage to the railway or other works, or else of demanding that the minerals be left unwrought, in which case compensation must be paid in respect thereof. The position appeared to be clearly defined until a decision in the well-known case of the L. and N.W. Railway Company *v.* Howley Park, etc., Company in 1911 introduced a disturbing element in that it laid down the principle that these Acts do not deprive the railways of their Common Law right to such lateral support as may be derived from mineral situated beyond the 40-yard limit, and that the working of mineral outside this limit in such a way as to cause damage to the railway renders those working the mineral liable to pay compensation for such damage. The whole question is at the present moment in an unsettled condition, and will probably need clarifying by further legislation.* At present it can only be said that it appears to be necessary to leave ample mineral unwrought so as to provide lateral support as well as to take into account the necessary vertical support for railways or other similar works.

The proper allowance for such lateral support is a very difficult problem, and one on which there are considerable differences of opinion amongst mining engineers.

The general principle † may be stated thus :—

Let Fig. 7 represent a vertical section through a bed of mineral and the overlying strata, C A Y being the surface of the earth and O B the roof of the bed of mineral, both supposed horizontal.

Let A O be the vertical trace of a plane at right

* Since the above was written, a Bill to deal with this matter has been drafted by the Department of Mines.

† See *Trans. Inst. Min. Eng.*, vol. lxiv., 1923, p. 257.

angles to the plane of the paper. Then in its natural condition the plane A O is in equilibrium under the horizontal thrust of the strata on either side thereof.

Let O X be the trace of a plane inclined at an angle X O B (a) equal to the angle of repose of the strata; then the horizontal thrust of the prism A O X slipping along that plane is practically zero, as would also be that of an infinitely narrow prism nearly coincident with the plane A O; between these two extremes there exists a plane, known as the plane of rupture, O Y, the

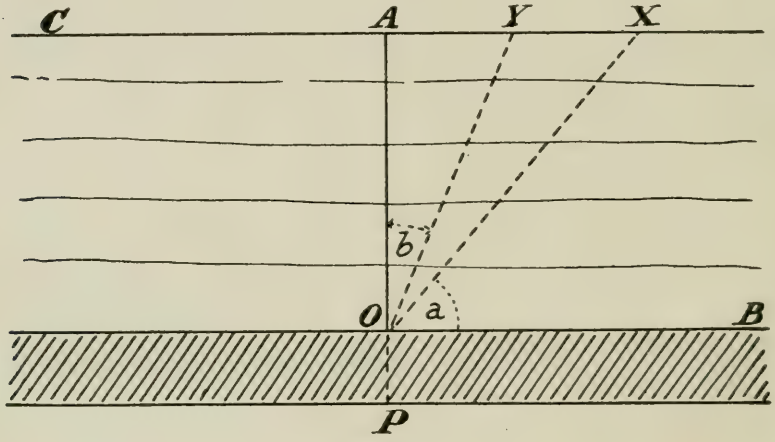


Fig. 7.

prism constituted by which exerts a maximum horizontal thrust, and it can be shown that the inclination of this plane to the vertical, the angle A O Y (b), is equal to $\frac{90° - a}{2}$ (Coulomb's theory of earth pressure).

If the whole of the strata to the left of the plane A O be supposed to be suddenly removed, so that there is no resistance to the motion of the plane A O towards the left, the entire prism A O X will give way and will slide down until the surface will form a plane corresponding to the angle of repose.

Next suppose that instead of the entire removal of the strata the bed of the mineral alone is removed to the left of the plane O P ; the strata, that were originally supported by the bed, being deprived of their vertical support, will now subside ; in so doing they will be broken and shattered to some extent, and will, therefore, occupy a larger volume than they did in their original position (Fig. 8). This direct subsidence will be less than the thickness of the bed removed, the difference being due to the increase in bulk of the broken-down strata.

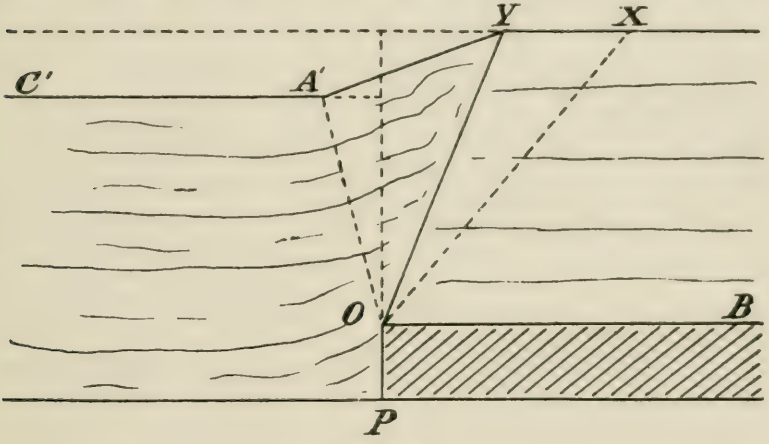


Fig. 8.

Apparently this increase is a maximum at first, and gradually decreases as the strata are again compressed under their own weight, but they never reach ultimately the same density or volume as they had originally. The plane A O will now be no longer in equilibrium, because the ground to the left of A O will be lower in position and lighter in density than the original ground to the right of A O. The plane A O will, therefore, be thrust into some such position as A′O, where it will reach a new position of equilibrium, when the thrust of the prism A′O Y has compressed the ground to the

left of A′O until its density is equal to that of the ground to the right of it, which has simultaneously expanded somewhat by partial shattering. As this is a case of resisted horizontal thrust, the maximum horizontal thrust of the prism A O Y naturally comes into play, and this, as already stated, is along the plane of rupture forming the angle *b* with the vertical. The extent of the indirect or lateral subsidence is usually spoken of as the "draw." It is sometimes defined as the horizontal distance A Y, but more often as the angle A O Y, the latter being the perferable method. Although for the sake of clearness the occurrences have been stated as though the process of vertical subsidence were complete before the draw commences, it must be understood that this is not what actually takes place ; vertical subsidence and lateral subsidence take place concurrently, the latter commencing as soon as any vertical subsidence shows itself. All that can be said is that in fact vertical subsidence is generally complete somewhat before lateral subsidence ceases altogether.

The above theory would indicate that the angle of draw should generally be half of 90° − angle of repose, though when the depth of the deposit is very great compared to the thickness of the bed of mineral removed, equilibrium may be reached before the draw has extended to the plane of rupture. If the surface of the ground, instead of being horizontal, is inclined, the position of the plane of rupture is thereby affected ; if the ground rises above the unwrought portion of the bed of mineral —*i.e.*, in this case to the right—the draw will be increased, and if the surface falls in that direction the draw will be correspondingly decreased. This effect will be barely noticeable unless the rise or fall is very considerable compared to the depth of the bed of mineral below the surface. When the strata are inclined, there

is a tendency for the thrust of the prism of rock to act down the inclined plane of the stratification, so that the inclination of draw is greater in the direction of dip and lower in the direction of rise than when the strata are horizontal. The exact effect of such inclination is by no means known, but most authors are agreed that once a certain dip is reached any increase of dip has no further effect. J. Dickinson * gives the limiting dip as 18°, T. A. O'Donohue † takes it at 24° ; the German authorities in Dortmund ‡ give the limiting angle at 35°. It will be noted that the limiting inclination at which the draw reaches its maximum is nearly double as great according to the last-named as according to the first-named of these authorities, and this fact alone would indicate that there is still very much to be learnt on this subject. As regards the definite effect that inclinations of the strata have upon the draw, ideas are still more vague. O'Donohue uses a purely empirical method. He takes the horizontal angle of draw in coal measures as 8° and adds two-thirds of the dip of the bed to this angle to get the draw on the rise side, and subtracts one-third of the inclination of the bed for the draw on the dip side ; for example, for a seam dipping 12° he would take the angle of draw on the dip side as 8° + 8° or 16°, and on the rise side as 8° − 4° or 4° ; he limits his formula to seams of 6 feet or less in thickness. The Dortmund authorities hold that for inclinations up to 15° the angle of draw will not exceed 15° ; that for inclinations between 15° and 35° it is equal to the dip of the bed, and for inclinations exceeding 35° the draw remains constant at that angle. Hause §

* *Trans. Manch. Geol. Soc.*, vol. xxv., 1897, p. 583.

† "Mining Formulæ," p. 246.

‡ "Zeitsch. f. Berg. Hütt. u. Sal. Wesen," 1897, p. 372.

§ "Zeitsch. f. Berg. Hütt. u. Sal. Wesen," 1907, p. 324.

gives three empirical methods and then averages them to get the method which he finally adopts ; he appears to think that the angle of draw is 20° when the bed is horizontal, but his data are far from clear.

In the Final Report of the Coal Conservation Committee (Cmd. 9,064), 1918, p. 64, the subject of subsidences is indirectly dealt with as follows :—

"The Railway Clauses Acts, and other Acts containing similar provisions with regard to the terms upon which coal may be required to be left unworked for the support of the surface should be amended so as to provide that the 'prescribed distance' within which the provisions referred to apply, shall be a distance from the railway (or other undertaking to which the Act applies), or any of the works connected therewith, equal to one-half of the depth of the workings below the surface level of the land at the place." In other words, the mining engineers upon this Commission consider that a safe angle of draw is 26° 35′, and presumably this is intended to apply to inclined strata as well as to horizontal ones. This angle is usually considered to err on the side of safety, and may be looked upon as a maximum. A number of careful measurements have shown that the average angle of draw in ordinary coal measure strata lying practically horizontal is between 5° and 15°, being lower the greater the proportion of sandstone and higher the greater the proportion of shale in the section. In Secondary strata—*e.g.*, in the Liassic rocks overlying the Cleveland ironstone—the draw is about 25°. In boulder clay or similar recent superficial formations, the angle of draw appears to vary within very wide limits between 20° and 35°. It will be seen that the data so far available give no very precise guidance as to the angle of draw, and accordingly it will always be best to be guided by local experience where

such can be obtained. If it is necessary to decide as to the width of barriers to be left in any case where no guidance can be obtained from previous workings, the following purely empirical formula may be used :—

Let b be the angle of draw on the rise side, b' on the dip side, measured to the vertical, and let d be the dip of the strata measured to the horizontal. Then

$$b = 10° + 40 \tan (d° - 5°)$$
$$b' = 10° - 20 \tan (d° - 5°)$$

Where d is less than 5°, b and b' are taken as 10°, and where d exceeds 30°, b and b' are taken as for 30°.

This appears to give results that agree reasonably well with experience in ordinary Coal Measure strata ; for Secondary and Tertiary strata substitute 20° for 10° in the above formula, and for clay, soil, etc., which is not stratified, 30° may be taken as a fair average figure. The depth of the seam below the surface multiplied by the tangent of the angle of draw will give the width of barrier to be allowed for suitable protection in such cases. There appears to be little doubt that in the case of thin beds of mineral, the width of the barrier may be taken as somewhat narrower, and for very thick seams as somewhat wider than the above formulas indicate, but there are no data available to enable the thickness of the seam to be introduced as a factor in the formula.

It will be seen that we are still very far from having arrived at anything like an accurate estimate of the requirements in any given case, and it will always be well to be guided by the local experience obtainable from previous workings whenever such is available.

In every case, all areas of mineral that have to be left standing for any of the above reasons must be deducted from the total surface area, in order to arrive

at the area underlain by workable mineral. This area can usually be got at with a reasonable degree of accuracy; the greatest difficulty is usually the determination of the third dimension—namely, the vertical thickness of the bed. As has already been pointed out, this dimension is never really known, and all that can be done is to estimate the most probable vertical thickness, using such data as are available and applying to them the principles of probability.

When the property to be valued is a mine which has already been worked to some extent, there is generally a certain quantity of mineral already blocked out, the calculation of which presents but little difficulty. It is rare that this forms any considerable quantity of the total mineral available; if a coal seam, for example, is being worked by longwall, there is hardly ever any of the coal blocked out, properly speaking; in a bord-and-pillar working a certain amount of coal may be standing in pillars, but the modern practice of following the whole workings as closely as possible by the brokens reduces this amount usually to insignificant importance. The real difficulty is always the determination of the quantity of mineral that is not blocked out, and the importance of existing workings lies mainly in the information obtainable from them as to the greater or lesser regularity of the bed and its probable average thickness. Again, the bed may have been worked in adjoining properties lying on one or more sides of the property to be examined, and if it can be inspected there, it is also possible to form an opinion as to whether it is liable to great variations in thickness or whether it appears to maintain a tolerably uniform thickness throughout; in the latter case a few boreholes will usually suffice to show whether it is safe to take an average value derived from all the available data, and

under such conditions it may be considered that a bed is sufficiently proved if no part of it is further away than, say, a quarter of a mile from a point where the thickness can be measured. Where the thickness of a bed is found to be very variable, boreholes or trial pits should be closer together, but no precise rule can be laid down. Necessarily the value of the mineral itself must be taken into account in determining how much proving should be done, because it may easily occur that the additional information gained by multiplying trial holes may not be worth the expense of putting them down. This is best determined by calculating the probable error of the average thickness as determined by the data at hand and then calculating how much it will cost to reduce this error down to any desired value; obviously the money value corresponding to any given probable error depends largely upon the nature of the mineral itself. It must be admitted that work on these lines savours somewhat of a counsel of perfection, and indicates rather the object to be aimed at than what is always practicable, for it cannot be too strongly insisted that economic considerations must always take the first place in valuing as in all other mining operations. It often happens that a mineral property has to be valued on admittedly insufficient data; thus a property on which little or no work has been done may be under offer with an option period so short that it is impossible to do any trial work within the given time, and yet it may be absolutely necessary to place a value upon it. The only possible method in such cases is to collect what information as to the thickness of the bed is obtainable and to trust to experience and geological knowledge to decide what may be taken as a fair probable figure for the thickness of the deposit. The prospective purchaser should in such a case always

be informed that the data are insufficient to enable an accurate estimate to be prepared, and that the estimate depends upon certain hypotheses which should always be clearly stated; he can then see for himself what is the measure of risk that he is running, and ultimately it comes back to the question whether it is worth the investor's while to take that risk or not. It is a risk that has constantly to be taken in mining matters, and it is precisely in these difficult cases that the experience and ability of the mining engineer form the decisive factor. It is obvious enough that each case must be considered on its own merits, and that no general rules at all can be laid down. Naturally enough every speculator, who is prepared to take risks such as are here indicated, expects and is entitled to expect a profit commensurate with the risk that he is running if his venture should turn out to be successful.

Alluvial deposits—*e.g.*, gold-bearing or tin-bearing gravels—form a special class of bedded deposits as far as valuation is concerned, because here the primary object is to determine not the probable quantity of material contained in the deposit itself, but rather that of the valuable mineral, such as gold or tin-stone, which it contains. The mass of the deposit itself is only of importance to the valuer inasmuch as it is one of the factors in determining the cost of getting the valuable mineral. It is evident that of two gravel deposits containing the same quantity of gold and equal in all respects except that one is twice as rich per cubic yard as the other and, therefore, contains only one-half as many cubic yards, this will be the cheaper to work, will leave a larger margin of profit, and will on this account be the more valuable deposit of the two. Before such deposits can be valued, they must be prospected, and this is done by sinking small trial pits or putting down

boreholes all over the deposit. The mode of laying out these trial holes varies according to the nature of the deposits themselves, whilst there is considerable divergence in the practice of different engineers. Mr. W. B. Middleton,* writing of tin-bearing alluvials, says "the writer's experience leads him to believe that no ground is properly prospected with less than one hole to the acre, and under ordinary circumstances the holes should never be more than 5 chains apart," and Mr. R. T. Hancock † points out that this object is best attained by placing holes at the apices of equilateral triangles of 3·4 chains side. Mr. F. J. Dennis ‡ states that in gold-bearing alluvials the trial holes are generally laid out in squares of 200 to 500 feet. Where small round shafts can be sunk cheaply, these are to be preferred. Cuts are then taken down the whole depth of the deposit, usually two cuts, about 6 inches wide, on opposite sides of the shaft. When boreholes are used, a casing must be driven down, 4 to 6 inches in diameter, and the whole of the material got out of the borehole then constitutes the sample. In boring there is always the danger that some of the heavy valuable mineral from the adjoining gravel may be drawn into the borehole and make the sample richer than it should be, and care is needed in order to get a true sample. The samples obtained in either way are very carefully washed, the resulting mineral weighed, and thus the average yield of the alluvial deposit is determined; in these calculations again the principles of probability can be applied with advantage.

It must not be forgotten that the above methods

* "Prospecting Tin Lands in Malaya." *Trans. Inst. Min. Met.*, vol. xiv., p. 302.

† *Op. cit.*, p. 325.

‡ "Examination of Dredging Properties." *Trans. Amer. Inst. Min. Eng.*, vol. xlii., 1911, p. 851.

determine the total quantity of workable mineral present in the given deposit or area. It is, however, practically impossible to extract the whole of the workable mineral, and allowance must be made for this fact. In all mining operations there is a certain amount of waste of mineral varying with the nature of the deposit, the nature of the mineral, and the method of working adopted; it does not always follow that the method which produces the largest proportion of mineral is necessarily the best to adopt. It quite often happens, especially with low-grade minerals, that the mineral itself is the cheapest article that the miner has at his disposal, and that it may pay him to sacrifice a certain proportion of this in order to work the remainder more cheaply, in order to simplify or improve the dressing operations, or to get his products to market with as little delay as possible. Such conditions must depend upon the individual cases, and it is quite impossible to lay down any rules; the experience of the mining engineer alone can decide such points.

The proportion of any bed of mineral that can in practice be economically extracted varies within very wide limits. A striking example is the Ten Yard or Thick Coal of South Staffordshire, where special methods of working must be adopted on account of the liability of the coal to spontaneous combustion. Mr. F. J. Meacham * has tabulated data showing that in many cases over 50 per cent. of the whole of the coal present is lost. The Royal Coal Commission of 1903 made many efforts to arrive at the waste of coal in Great Britain, but the subject was found to be attended with great difficulty owing to the excessive variations in different cases. Thus Mr. J. Higson †

* Royal Coal Commission, 1903, First Report, p. 273.

† Royal Coal Commission, 1910, Second Report, p. 53.

states that in his experience the wastage in the ordinary working of the mine ranges from 1 to 25 per cent. The total figures collected by the Commission indicate a waste throughout the country of about 20 per cent., of which about one-third is due to coal left in barriers and for the support of buildings, etc., and the remainder to natural causes, such as faults, etc., and to loss in actual working. For the present purpose, it is only the latter 13 per cent. that should be taken into account, the former figure having been dealt with independently. The Coal Conservation Committee's Final Report (p. 10) appears to consider that "pit wastage," which may possibly include coal left for support, amounts to about 15 per cent. Generally speaking, the loss in working would seem to lie between 5 and 15 per cent., depending upon the conditions already enumerated, and is sometimes even lower than the former figure.

It must further be remembered that, in valuing coal royalties, it is quite usual for the lessor to allow a certain amount of coal to be worked free of royalty for colliery consumption. The Coal Conservation Committee's Report (p. 50) gives 6·8 per cent. of the total output as the average colliery consumption in Great Britain.

In working ironstone underground, the waste apart from ironstone left to support buildings, etc., is not at all great; it is usually estimated in Cleveland that it is not necessary to leave more than 55 per cent. of the total ironstone in pillars, provided that these are properly proportioned, to support any ordinary buildings, railway, etc.; the actual loss in working ironstone, such as that of Cleveland, is very small, probably only a few per cents.; an ironstone that makes very much fines, as is the case with some of the Northamptonshire

ironstone, is liable to produce a greater waste. In some parts of the Northamptonshire field it is the custom to load ironstone with forks, the tines of which are approximately 1 inch apart, and to throw the small back into the waste; this may produce a loss of 10 to 15 per cent. of the total ironstone present.

The probable available volume of mineral in a given deposit or property is estimated in accordance with these general principles. It then remains to convert this figure into the corresponding tonnage. If the volume is obtained in cubic feet, the basis for calculation is the weight of a cubic foot of water—namely, 0·5576 cwt. or 0·02788 ton. In this country the basis for calculation is often a horizontal surface of so many acres, the vertical thickness of the deposit being determined in feet or inches; one acre of water 1 foot thick weighs 1,215 tons, and an acre of water 1 inch thick about 101 tons. These figures must be multiplied by the specific gravity of the mineral in order to obtain the required weight of mineral per acre per foot or per inch thick as the case may be. The specific gravity of coal ranges from 1·1 to 1·6, the average of most bituminous coals being about 1·3. An acre of such coal would thus weigh about 130 tons per inch thick per acre; in rough calculations the yield of a coal seam is often taken at 100 tons per inch per acre, thus making a liberal allowance for faults, coal left for support, coal lost in working, etc. In the same way, and for similar approximate estimates, the weight of ironstone is often taken roughly as 2,000 tons per foot thick per acre. The following table gives the specific gravities and the weights per cubic foot of a number of minerals that are most generally worked in bedded deposits:—

Mineral.	Specific Gravity.	Weight per cub. ft. Cwts.
Coal,	1·1 to 1·6	0·61 to 0·89
Scottish Oil shale,	1·75	0·98
New South Wales Kerosene shale,	1·1 to 1·3	0·61 to 0·73
Fireclay,	about 2·5	1·40
Ganister,	about 2·6	1·45
Limestone,	2·45 to 2·6	1·37 to 1·45
Rocksalt,	2·1 to 2·26	1·17 to 1·26
Cleveland ironstone,	2·65	1·48
Northamptonshire ironstone (blue),	2·11 to 2·26	1·18 to 1·26
Northamptonshire ironstone (brown),	2·00 to 2·11	1·11 to 1·18
Lincolnshire ironstone,	about 2·12	1·19
Red Hæmatite, Wabana, Newfoundland,	4·3 to 4·5	2·40 to 2·51
Clayband ironstone,	2·9 to 3·2	1·62 to 1·78
Blackband ironstone,	about 2·5	1·39

NOTE.—To convert cwts. per cubic foot into tons per foot per acre, multiply by 2,178.

VEINS.

Here the data are usually less well determined than in the case of beds, whilst owing to the fact that veins are, generally speaking, much more irregular in their character than are beds, more exploratory work is needed in most cases. On the other hand, however, owing to the generally steep dips of veins, and to the fact that they frequently traverse the harder rocks, and in many instances are met with in mountainous regions, their outcrops are frequently well marked, and can be traced on the surface. In this way one of the required dimensions—namely, the length—can often be readily ascertained with comparative accuracy. The depth to which a vein will go down is in practice never known, and must be assumed

on the basis of such information as can be obtained from the geological structure of the country and the nature of the vein itself. Generally the vein is proved by one or more shafts, adits, or cross-cuts, which intersect the vein at some distance below its outcrop, and in the majority of cases there will be a shorter or longer drive or drives upon the vein itself, so as to enable its characteristics to be studied. It is often assumed in such cases, where there is no change in the country itself, and where the vein shows no sign of pinching at the lowest point at which it can be seen, that the vein may be expected to hold down at least to a vertical depth below the lowest exposure equal to one-half of the depth below this and the outcrop, though it is rarely safe to assume an extension of more than 100 feet below the lowest known working. It may be assumed that the vein is likely to hold down to whichever of the depths as above determined is the lesser, and that it gradually wedges out to zero at this depth, so that the cross-section of the portion assumed to exist below the lowest point at which the vein is visible would be considered as having only one-half of the normal thickness of the vein, or to put it from a different point of view, only one-half of the "weight" is assigned to the probability of the vein holding its normal width below the last point where it can be seen as is assigned to the probability of its maintaining its normal width above that point. Such assumptions must be used with caution, though it often happens that there are no other data available. It cannot too often be repeated that estimates based on such slender evidence demand the highest technical and mental qualifications on the part of the mining engineer, who is called upon to make sound deductions from such facts as he is able to gather, reviewed in the light of his geological knowledge and

experience. It is easy enough to assume the attitude of declining to take into account any mineral below the lowest visible exposure, but obviously anyone working on this principle must necessarily under-estimate the tonnage that does in actual fact exist. The engineer must do his utmost to steer a fair course between the Scylla of under-estimating and the Charybdis of over-estimating; the former may sacrifice the opportunity of acquiring a valuable property, the latter may lead to very serious losses; of the two errors, the first-named is no doubt the less grave; the engineer must remember that "it is easier to keep out than to get out," and must, therefore, exercise due caution, though at the same time he must strive to avoid excessive timidity. It need hardly be added that each such problem must be solved in the light of its own data, and that general principles can afford no guidance whatever.

Often a distinction is made between so-called proved ore and probable ore; proved ore, also spoken of as ore in sight, but preferably designated ore blocked out, is usually taken to be ore which has been exposed on four sides—*i.e.*, if a level is driven on a vein at some depth below the outcrop, and a couple of shafts are sunk in the vein from the outcrop to this level, the block of ore comprised between the outcrop, the level, and the two shafts would be spoken of as proved ore. The word "proved" is here used in the miner's sense of tested, because it is obvious that there is no proof at all that the whole of this block does in fact consist of ore, so that even where a certain quantity of such ore has been blocked out, it can only be looked upon as ore regarding which the probability of its existence in fact is much higher than that which attaches to ore not so blocked out, although in every case the pro-

bability falls short of a certainty. The determination of the quantity of ore blocked out is a task merely for the surveyor, whereas the estimation of the ore not thus proved, as already pointed out, calls for the highest degree of experience and geological knowledge on the part of the mining engineer. It must, however, be insisted that the distinction between these two categories is more apparent than real, and is one of degree and not of kind.

The probable quantity of mineral in a vein is usually calculated as in the case of a bed, by considering the vein as a thick sheet, the thickness of which is taken to be the average width of the vein as determined from the data available in the same way as in the corresponding case of the thickness of a bed. As already pointed out, the length is generally known with some degree of certainty from the outcrop, and the depth is either known or assumed. If it is required to determine the probable quantity of mineral, not in the entire length of a vein, but only in such length as is included within a certain area, say, for example, a concession of definite length, it will be found that the portion of the vein thus included frequently takes the shape of a rectangle, the longer side of which is, as a rule, parallel to the average strike of the vein. In the majority of cases the concession is assumed to be bounded by the vertical planes, the horizontal traces of which correspond to the concession boundaries set out on the surface, and only so much of the vein belongs to the concessionaire as lies within these four vertical planes. An exception to this limitation is met with in the case of certain Claims, such as those in the United States, subject to the so-called "Law of the Apex," where a claim gives the right to follow any vein that outcrops within the superficial area of the claim down along its dip to any desired

depth, although in so doing, it passes beyond the vertical plane which includes the side line on the dip side ; in these claims the length of the vein, as limited by the end lines, without any lateral limits, is conveyed by the grant of the mining claim. In the more usual case, where the mineral deposit granted is limited by vertical planes, it is necessary to determine the average dip of the vein ; let this be $a°$ to the horizontal ; if the distance of the outcrop from the boundary of the concession towards which the vein dips be d, the depth of vein included within the concession is $d \tan a$. As in the corresponding case of beds, the true width = the horizontal width $\times \sin a$. In the same way, except for the above determination of depth, it is unnecessary to know either the average dip or the true width of the vein if its horizontal width is known. The volume of mineral is equal to the horizontal width $\times$ vertical depth $\times$ length, and this is usually the easiest way of performing the calculation, because the horizontal width is generally readily ascertainable by prospecting operations, so that, if the vertical depth of the vein is h, its length l and its horizontal width m, the volume of mineral included in it is represented by $h\,l\,m$. Strictly speaking, these dimensions should be referred to the median plane of the vein, but this refinement is negligible save in the exceptional case where the width of the vein is very great and the area involved is small. If the strike of the vein be parallel to a side line of a concession area, and l is now the length of the concession or the distance between the end lines, the distance of the outcrop from the side line towards which it dips being, as before, d, and the inclination to the horizontal $a°$, the volume of mineral contained in the concession is $l\,m\,d \tan a$. If the strike of the vein be not parallel to the side lines, but makes an angle θ with them, if

d be the shortest distance between the outcrop and the side line towards which the vein dips, measured along one of the end lines, the distance of the outcrop from the same side line, measured along the other end line being $d_1 = d + l \tan \theta$ (see Fig. 9, where A B is the plan of the vein), the corresponding vertical depths at which the vein passes out of the concession are respectively $d \tan \alpha \cos \theta$ and $(d + l \tan \theta) \tan \alpha \cos \theta$; the volume of mineral in that portion of the vein that is included within the concession is then

$$\frac{\tan \alpha}{2}(2d + l \tan \theta)\, l\, m, \quad \text{or} \quad \tan \alpha \left(\frac{d + d_1}{2}\right) l\, m.$$

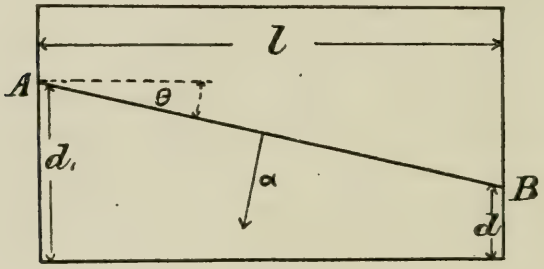


Fig. 9.

If the surface of the ground is very undulating, the vertical depth must be taken as the depth below the mean level of the surface. In simple side-long ground having a tolerably uniform slope this can be calculated if the angle of slope is known, but when the surface is irregular it is best to plot a vertical section and to determine the mean level from the drawing, or to determine by planimeter or otherwise the area of the projection on a vertical plane.

Special attention must be given to those cases where the vein is not uniformly mineralised, as, for example, the lead veins of the Mountain Limestone; in these cases only so much of the depth of the vein as occurs

within the productive sills must be allowed to enter into the calculation. If the vein is barren or not payable except in certain well-defined shoots, only the horizontal length of these shoots must be taken into account, and not the total length of the vein. If the horizontal length of the pay-shoots is known, their pitch can be disregarded, on the same principle as applies to calculating the width of the vein. As a good example of the necessity for taking only pay-shoots into account, the veins of Spathic ore in the Brendon Hills, Somerset, may be quoted. These have been quite extensively worked in the past, so that their structure is fairly well known. They dip steeply in strata of Devonian Age; for the greater part of their length they are relatively narrow veins filled with iron-stained quartz containing very little true iron ore, but they widen out in places into shoots from 200 to 1,000 feet in length, often of a fair width, filled with spathic iron ore of good quality. The main lode has been traced on the surface for some 37,000 feet, but underground workings have shown that the workable pay-shoots in this distance are less than 3,400 feet in length, so that only 9 per cent. of the total length is payable, and in estimating the quantity of ore it is this figure alone that should enter into the calculation. The fact that the values in a mineral vein are often concentrated in pay-shoots, the rest of the vein being comparatively or absolutely worthless, adds greatly to the difficulty of valuing veins unless a very large amount of work has been done upon them; if a vein has been driven upon for a short distance only, it is usually impossible to decide what proportion of the total length is correctly represented by the section that has been opened up both in respect of width and of quality. Under such circumstances local knowledge is invaluable, and in all cases great attention should be

paid to any change or indication of change in the country rock, because this is so often accompanied by corresponding changes in the vein itself.

The proportion of workable mineral that can be obtained from veins is in most cases a large one, there being many examples where the wastage is practically nil; the yield is always greater with over-hand than with underhand stoping, though it is not possible to give any generally applicable indication of the amount of such difference. The existence of "horses" or inclusions of barren rock in the vein must always be taken into account. In some cases, especially where the dip of the vein is relatively flat, where the hanging wall is weak and where timber is expensive, pillars of vein-stuff may be left in the stopes in order to support the hanging wall. The mining engineer naturally endeavours to leave such pillars in the poorer parts of the vein, but this is not always possible. In any case where it is probable that this method may have to be resorted to, a reduction should be made for the mineral thus left standing. On the other hand, if a vein is approximately vertical, has hard walls, and can be worked by such a system as magazine mining, practically the whole of the payable contents of the vein may be looked upon as recoverable ore.

MASSES.

Owing to the excessive irregularity of shape of the deposits classed under this head, it is extremely difficult to formulate any rules at all for determining the volume of mineral that they may be expected to contain. Generally speaking, an elaborate scheme of trial workings or borings is needed before even an approximate estimate can be made. As an example, the method used in estimating the probable quantity of ore is the masses

of red hæmatite on the Misabi Range may be quoted * :— "in these flat Misabi ore-bodies composed of a number of layers of large area and relatively small thickness, information on vertical lines is of paramount importance. When a piece of ground has been properly explored drill holes in the ore-body are usually 200 feet apart . . . the first step in making an estimate is to study and properly map the exploration data. Maps for operating purposes are usually made to a scale of 50 feet = 1 inch, or 100 feet = 1 inch. For estimating and valuing purposes a scale of 200 feet = 1 inch is often used. After a plot is prepared showing the location and elevation of drill holes, test-pits, drifts, etc., cross-sections may be made of the ore-body on a vertical and horizontal scale of 40 feet = 1 inch . . . the outline of the ore-body is established from the drill holes. Beyond the outer or limiting drill holes there is some ore which is estimated as follows :—On each cross-section the ore is followed beyond the last hole in ore to the point where the section indicates a thickness of ore equal to half the thickness shown in the drill hole. For estimating purposes this point is considered the limit of the ore-body. There are two such points to each section. By getting these points from all the sections the area of the enclosed layer of ore is found."

Similar methods have been described by Messrs. Wolff, Derby, and Cole in *Mining and Metallurgy* for September, 1922 ; according to these writers, the limiting edge of the ore body is determined as above or else by drawing an outline at an arbitrary distance with a maximum of 100 feet outside the holes in ore. "Another method is to assume ore to extend 100 feet beyond the limiting holes, if the depth of ore in the hole is 50 feet

* "Iron Mining in Minnesota," by Charles E. Barneveld. The Minnesota School of Mines Experiment Station, *Bulletin*, No. 1, 1912, p. 35.

or more, to extend 50 feet beyond if the depth is 25 to 50 feet, and to extend 25 feet beyond if the depth is less than 25 feet." The method by cross-sections as given above is admitted to be the more accurate, but it is stated that many engineers, after determining the area of the deposit by one of the three methods given, simply multiply this area by the average depth of the ore in feet in order to obtain the volume of the ore present. In order to convert volume into tonnage, factors are used which have been obtained by experience, ranging from 10 cubic feet to 18 cubic feet to the ton. On the Gogebic Range ore assaying over 60 per cent. of iron in the dry is estimated at 10 cubic feet to the ton, and ore below 57 per cent. of iron in the dry at 11 cubic feet to the ton; 10 per cent. is deducted to cover rock that may be included in the ore.

Somewhat similar methods are followed in the irregular deposits of zinc blende in South Western Wisconsin,* where the limits of the ore bodies, which are relatively small, are determined by drilling, almost invariably by churn drills. "The locations of the drill holes are platted on a map, and the boundary of the ore body is considered as a line joining the outside holes that are in ore. The included area is measured. The arithmetical total of the thicknesses of minable ore shown by each hole in this area is divided by the number of holes. The result gives an approximate figure for the average thickness of the ore body. . . . The area in square feet multiplied by the thickness in feet and divided by the average number of cubic feet of mineralised rock per ton gives the total tonnage of

* Wisconsin Geological and Natural History Survey, *Bulletin*, No. xli., "A Study of Methods of Mine Valuation and Assessment (1914)," by W. L. Uglow, p. 13.

reserve ore. In these computations the figure 12 is taken as the average number of cubic feet per ton."

Other examples of such procedure are to be found in the porphyry copper deposits in Arizona, the following quotations showing the methods employed:— "the general plan on the ground now owned by the Inspiration Consolidated Copper Co. (Miami) has been to test the ore zone and define its limits by holes placed 200 feet at the intersections of a regular system of rectangular co-ordinates. . . . On outlying properties, where the presence of ore in commercial quantity has not been demonstrated, scout holes are put down with less attention to regular spacing and alignment." *

Again, "the ore-bearing area at Ray was explored by churn drilling, the holes as a rule being situated at the intersections of rectangular co-ordinate lines 200 feet apart. The Ray Consolidated Copper Co. tested in this way an area of about 185 acres. The total number of holes drilled is 353, and their average depth is 418 feet. . . . The average cost of drilling at Ray was between $2 and $3 a foot." †

It more often happens that estimates have to be prepared before any such complete exploration has been possible, and it is not infrequently the case that there are just three dimensions of a mass that are known, namely, its maximum length, its maximum width and the maximum depth to which it extends; for the sake of simplicity it may here be assumed that these three dimensions have been determined at practically right angles to each other. The largest possible volume of a regular figure, to which these axial dimensions could

* "The Copper Deposits of Ray and Miami, Arizona," by Frederick Leslie Ransome. U.S. Geological Survey, Professional Paper 115 (1919), p. 96.

† *Ibid.*, p. 101.

correspond, would be given by a prism; if the length of the three axes are a, b and c, respectively, the volume of the prism would be $a\,b\,c$. The smallest possible volume of a regular figure would be that given by a double-ended pyramid, of which the base is the rhombus having two of the axes for its diagonals, whilst the height of each pyramid is one-half of the third axis. The volume of this figure would be $\frac{a\,b\,c}{3}$; if it were unknown to which of these extreme forms the actual form approximates, the average of the two, or $\frac{2\,a\,b\,c}{3}$, might be looked upon as the most probable figure for the true volume; this figure is, however, too high, because in fact no mineral deposit approximates to the prismatic form, whilst there are some that might be roughly represented by the double pyramid. On the other hand, a good many masses, particularly deposits of a lenticular habit, affect forms that approximate to an ellipsoid; the volume of a regular ellipsoid having axes as above is $\frac{\pi}{6}\,a\,b\,c$, or $0{\cdot}5236\,a\,b\,c$; upon the whole, where nothing is known as to the shape of a mass, and only three axial dimensions have been proved, it may be considered prudent to take as the most probable figure for its volume the expression $\frac{a\,b\,c}{2}$, which would appear to be, if anything, somewhat below rather than above the most probable value.

In all other cases the method to be adopted must depend essentially upon the amount of information available; often a considerable portion of a mass is exposed, and then the ordinary methods of surveying are applicable. An excellent example is to be found in the determination by Mr. R. L. Jack of the volume

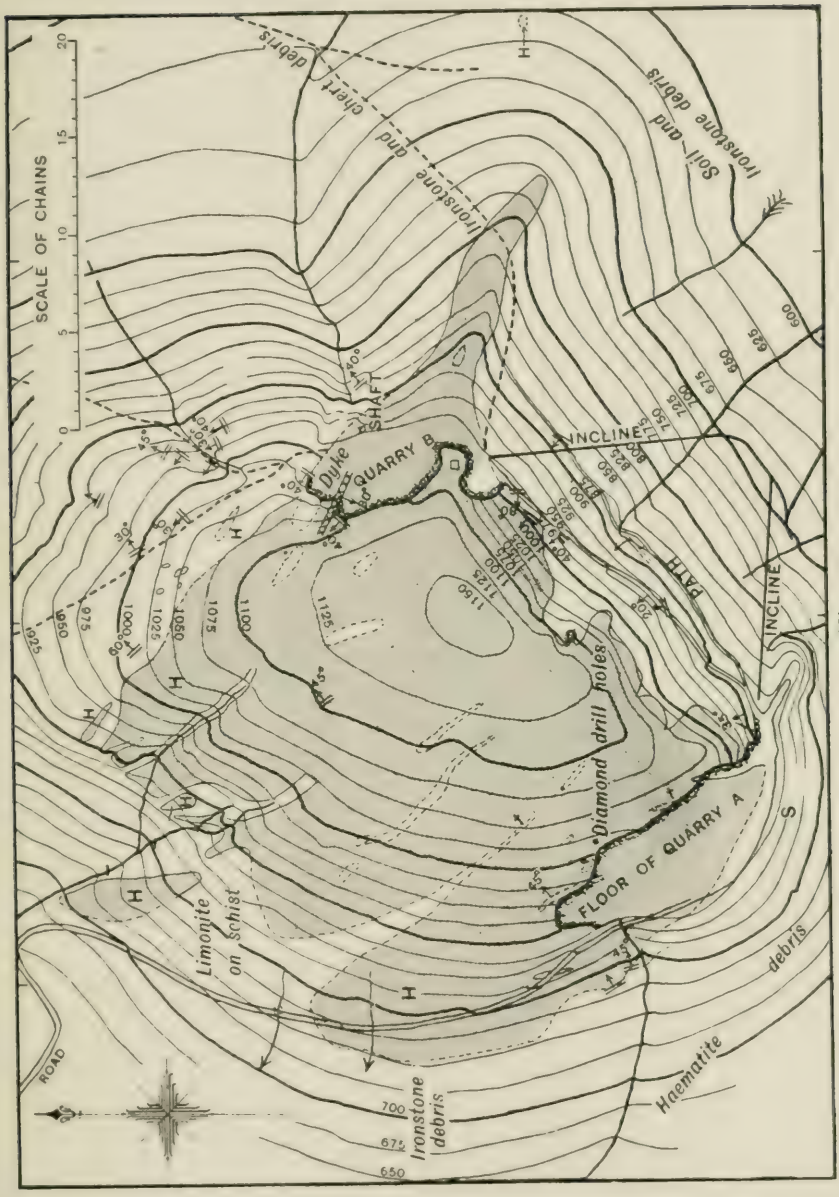

SCALE OF CHAINS
0
5
10
15
20
Limonite on Schist
Ironstone debris
Haematite debris
Diamond drill holes
FLOOR OF QUARRY A
QUARRY B
Duke
SHAFT
INCLINE
INCLINE
PATH
ROAD
Ironstone and chert debris
Soil and Ironstone debris
700
675
650

Fig. 10.

of the large mass of magnetite known as the Iron Knob,* in South Australia. Mr. Jack surveyed the mineral and plotted contour lines round it at a vertical distance of 25 feet apart, as shown in Fig. 10, reproduced by permission; he was able to assign a reasonable value to the shape of the concealed side of the mass, which he looked upon as a plane surface; he determined the area of the mass included by each contour line and the assumed wall of the deposit, and by this means was able to form a very reasonable estimate of the ore-body down to valley level. The outcrop area includes a certain amount of barren rock, which by measurement amounted to 8·9 per cent., and it has been assumed that the same proportion holds throughout the mass, so that 9 per cent. of the cubic content as above arrived at was deducted for waste.

This method may be contrasted with a case in which the data available are of the scantiest nature, as, for instance, the method adopted by Mr. Bernard Smith † in estimating the probable ore reserves in the deposits of red hæmatite presumed to exist in the unproved portion of the concealed iron ore fields south of Egremont, Cumberland; he determines the area of this ore field, and proceeds:—"provided that ore occurs as freely in this area as in the proved field to the north, there should be a reserve of about 50 million tons of ore." It is obvious that under the circumstances no other method of obtaining an approximate estimate is available, but that at the same time the probable error affecting the estimate in the latter case is many times

* Geological Survey of South Australia, Department of Mines, *Bulletin*, No. 9, p. 36.

† Memoirs of the Geological Survey. Special Reports on the Mineral Resources of Great Britain. Vol. VIII.—Iron Ores: Hæmatites of West Cumberland, Lancashire, and the Lake District.

greater than in the former one, although no numerical value can be assigned to the magnitude of the probable error.

When masses of magnetite occur which do not outcrop, they can be located and some idea of their magnitude obtained by means of the dipping needle as used in Sweden, but it is inadvisable to rely upon data obtained in this way without confirming the indications by means of boreholes.

As a general rule, masses tend to greater uniformity in richness of mineral content than do veins, and some of them quite equal beds in this respect, although this statement is liable to a good many exceptions ; obviously masses which owe their mineral wealth to disseminated minerals are much more likely to vary than an ore-body formed by segregation.

The waste in working masses varies within the widest possible limits. Where a mass can be quarried or worked underground by such a method as magazine mining, the loss is generally very small, and may even be negligible. On the other hand, a large irregular body of soft mineral, like some of the soft red hæmatite of Cumberland and Lancashire, which is worked by some system of caving, is liable to quite serious losses, which may even amount to 25 per cent. of the deposit.

In the zinc mines of South-Western Wisconsin,* where the ore occurs in relatively flat-lying irregular deposits in limestone and dolomite, approximately 10 per cent. of the volume of crude ore (ore and country rock in place) in a deposit is left in the mine as pillars and not recovered.

In the Hæmatite Mines of the Gogebic Range, Min-

* Wisconsin Geological and Natural History Survey, *Bulletin*, No. xli., "A Study of Methods of Mine Valuation and Assessment (1914)," by W. L. Uglow, p. 13.

nesota, " 10 per cent. reduction usually is allowed for mining loss and 10 per cent. for rock encountered in the ore." *

The following table gives the specific gravity and weight per cubic foot of minerals most generally occurring in veins and masses :—

Mineral.	Specific Gravity.	Weight per Cub. Ft.
		Cwts.
Quartz,	2·5 to 2·8	1·39 to 1·56
Calcspar,	2·5 to 2·7	1·39 to 1·50
Dolomite,	2·8 to 2·9	1·56 to 1·62
Fluorspar,	3·0 to 3·2	1·67 to 1·78
Apatite,	2·9 to 3·2	1·62 to 1·78
Iron pyrites,	4·8 to 5·2	2·68 to 2·90
Mispickel,	6·0 to 6·4	3·35 to 3·57
Pyrrhotite,	4·4 to 4·7	2·45 to 2·62
Magnetite,	4·8 to 5·2	2·68 to 2·90
Red hæmatite,	4·5 to 5·3	2·51 to 2·96
Brown hæmatite,	3·4 to 4·4	1·89 to 2·45
Spathic ore,	3·4 to 3·9	1·89 to 2·17
Pyrolusite,	4·3 to 4·9	2·40 to 2·73
Copper pyrites,	4·1 to 4·3	2·29 to 2·40
Chalcocite,	5·5 to 5·8	3·07 to 3·23
Malachite,	3·7 to 4·0	2·06 to 2·23
Galena,	7·3 to 7·7	4·07 to 4·29
Cerussite,	6·4 to 6·5	3·57 to 3·62
Anglesite,	6·1 to 6·4	3·40 to 3·57
Zinc blende,	3·9 to 4·2	2·17 to 2·34
Calamine,	4·0 to 4·5	2·23 to 2·51
Stibnite,	4·5 to 4·6	2·51 to 2·56
Cinnabar,	8·9 to 9·0	4·96 to 5·02
Cassiterite,	6·4 to 7·1	3·57 to 3·96
Wolfram,	7·1 to 7·6	3·96 to 4·24
Scheelite,	5·9 to 6·1	3·29 to 3·40
Native gold,	15·6 to 19·5	8·70 to 10·87

* J. F. Wolff, *Mining and Metallurgy*, September, 1922, p. 46.

CHAPTER IV.

DATA FOR VALUATION (*Continued*).

When the probable total amount of mineral that can be produced by a given property has been determined, it next becomes necessary to determine the probable rate of exhaustion of that quantity, a consideration of very great importance in valuation, for it is obvious that the more rapidly a given quantity of mineral can be worked out, the sooner will the money paid for it be recovered, and, therefore, the greater will be its actual value at the time of commencing operations. The life of a given mineral deposit or property is obtained by dividing the probable total quantity of mineral that it can produce by the probable annual production. This latter depends on a number of considerations, which are often purely economic in character. In principle there is for every given deposit or property a definite rate of exhaustion that will yield the maximum profit; it is readily seen that this must be so by considering quite extreme cases. Assume that the mineral property consists of a coal royalty of 1,000 acres, estimated to contain 5,000,000 tons of coal. Theoretically it would be possible by sinking a sufficient number of shafts over the whole property to work out the whole area in one year, or again by working only 20,000 tons a year it might be made to last two-and-a-half centuries. In the first case the operation would result in loss, because the capital outlay would be far greater than any possible profit to be obtained by the sale of the coal,

and in the second case there would also be a loss, because interest on capital and standing charges would exceed the profit, if any, on the coal wrought. Between these two extremes there must lie a particular rate of extraction that would yield the maximum rate of profit, but in practice it will be found that within certain quite wide limits the profits to be expected are not greatly affected by the rate of extraction. Thus, in the above case, the total profits obtainable would probably be affected very little by making the life of the property anything between 10 and 20 years. Frequently it is the needs of the mine owner or the capacity of the market for absorbing the product that will determine the annual rate of production, and it quite often happens that the producing capacity of a mine is, during the life of the mine, increased many times above that which was originally intended.

The rate of exhaustion of a mineral deposit depends, not only on the tonnage available, but upon the general conditions of the deposit in all its characteristics. The relative importance of the richness of a mine and of its life, for example, may vary considerably according to the nature of the mine itself. The point has been very well put by Mr. James R. Finlay in a comparison in this respect between gold mines and iron mines*:— As gold has an unlimited market, "the prime uncertainty is the ore itself . . . the amount of ore is usually the principal subject of uncertainty. Once mining is successfully established the cost is practically determined, so that almost the whole matter of valuation is the life of the mine." In iron ore mining the circumstances are entirely different, the supplies of ore being in many cases (especially in the United States) very

* "Valuation of Iron Mines." *Trans. Amer. Inst. Min. Eng.* (1913), vol. xlv., p. 282.

large indeed; under these conditions "the question is not primarily the supply of ore. The main questions are decidedly the price at which it can be sold and the amount that can be marketed each year . . . an error of 10 per cent. in estimating the average price of an ore will often make far more difference in the valuation than any possible mistake as to the life of the mine." In mines of an irregular type, where there is considerable uncertainty as to quantities, some engineers, of whom Mr. G. A. Denny * is one, consider that in the valuation of a mine the actual value should be calculated on the basis of repayment of the entire capital in not over ten years, irrespective of the life of the mine itself.

Generally speaking, for every mineral property there can be estimated a minimum rate of production, which forms the basis of the calculation of dead rent. In making a valuation, it is assumed that the mine is to be laid out with the object of producing a definite annual output, plant being provided which is presumed to be sufficient to deal with the product in question. The projected output is, however, never reached immediately that mining operations commence. In every mine there is always a period of preparatory work such as sinking shafts, driving adits, opening up the deposit, erecting machinery, building houses, and collecting workpeople, during which no mineral is raised, or, at any rate, none at remunerative rates; some may be obtained during the process of development, but this is incidental to the development, and is not the object of the operation. The costs incurred during this period are all considered as forming part of the capital outlay, the value of any mineral obtained being set against the same. Next comes the period of getting mineral, ac-

* "The Commercial Aspect of Rand Profits." *Mining Journal*, August 24th, 1907, p. 242.

companied by intensive development, during which the output is progressively increased until the full expected tonnage is reached. This output is maintained, with the inevitable fluctuations due to the hazards of mining, during the working life of the mine, or it may be deliberately increased or decreased, and finally during the last few years the output gradually falls off until there is too little mineral left to admit of profitable extraction, when the mine is considered as worked out. In valuing a mineral property all these successive stages must be taken into consideration; their probable duration can only be determined by experience with due regard to all the attendant conditions. If the object of the valuation is the lease of a mine already opened and in operation, the lease being for a term of years which is less than would be required to work out the probable amount of mineral contained in the mine, the proposition assumes a much simpler form, the life of the asset being simply the duration of the lease, whilst it is usually assumed that the full annual output will be kept up throughout it. If the lessor's interest has to be valued, it must be noted that most leases give the lessee the right to relinquish his lease, usually at stated periods, upon giving a stipulated notice; the possibility that the mine may become unprofitable or that the lessee may for some other reason desire to stop the operations or to cancel his lease must be taken into account; the measure of probability of such an occurrence must be estimated and the risk allowed for in the valuation.

The annual income derivable from the mineral property requires different treatment according as it is either the product of the ownership of mineral rights or as it is the result of mining operations, the former being obviously far less speculative in its nature than the latter. As has already been indicated, when a mineral

property is purchased for the express purpose of being worked by the purchaser, the valuation is best made as of a mining operation pure and simple, the royalty that would otherwise be payable on the unsevered mineral being treated as so much additional profit which is earned by that portion of the capital outlay which is allocated to the acquisition of the mineral rights.

In dealing with mineral rights by themselves, the only income is the royalty rent paid in respect of the tonnage of mineral worked within the year. As a general rule, this rent consists in part of a dead rent and in part of a royalty based upon the tonnage extracted; the former is a fixed annual amount, the latter is likely to fluctuate from year to year. There is thus much less risk attached to the former income, and, as already pointed out, this may be looked upon as a certain revenue, subject only to the possibility of the person or corporation working the minerals relinquishing; the possibility of default in payment need hardly ever be considered, because in such cases the owner can recover any indebtedness to himself out of the plant and assets of the mine and re-enter into possession of his original rights; he will thus, as a rule, incur no loss, and runs only the risk that the property may lie unproductive for a time until he finds a new lessee. If the cause of the default is due to the fact that the mine owner has conducted his operations badly, if his methods are wrong, or if his administration is defective, it is probable that the property will not long remain unproductive on the proprietor's hands; if, on the other hand, it is due to some inherent defect in the mineral itself, if it is too poor or too badly situated to be workable at a profit, its value for the time being is nil, and anything that the owner has received in the way of royalty is pure profit. The dead rent may, therefore, be treated as practically

certain income. The tonnage royalty must be calculated upon the probable amount of mineral that will be worked annually, and will fluctuate according to the risks of the market ; its amount is, therefore, far less certain than the former, and capital invested in it is, therefore, entitled to a commensurately higher rate of interest.

As has been stated (p. 11), dead rent may be of two kinds ; it is generally in the form of a merging dead rent—that is to say, that the lessee is allowed in consideration of the payment of dead rent to work free of royalty such a quantity of mineral as at the agreed royalty is equivalent to the dead rent. Sometimes, however, it takes the form of an additional dead rent—that is to say, a dead rent which has to be paid over and above the tonnage royalty without any deduction. Where merging dead rent is agreed upon, the lessor not infrequently gives an additional privilege—namely, that if in any year the amount of dead rent paid exceeds the royalty value of the mineral actually extracted, the lessee shall be allowed to carry forward to subsequent years the amount by which the tonnage royalty falls short of the dead rent and to work free of royalty as much additional mineral as may correspond to this sum in a subsequent year or years. For example, if the royalty on a given mineral is 6d. per ton and the merging dead rent is £1,000 per annum, the lessee will be charged no royalty in respect of the first 40,000 tons worked, and the royalty of 6d. per ton is only charged on a tonnage exceeding that figure. If in any year he has only worked 30,000 tons, the royalty on which amounts to £750, he will be short by 10,000 tons, equivalent to £250 in royalty, and will accordingly be allowed to work 10,000 tons over and above his first 40,000 tons in the following year without payment of any royalty beyond the dead rent, or he may be allowed to work 5,000 tons in each of the two

following years in the same way. This privilege is usually spoken of as permission to work "shorts"; it is usually restricted to a definite number of years, frequently from three to seven, or it may extend throughout the whole period of the lease. This power to work shorts gives rise to some of the most difficult problems in mineral valuation; if, for instance, the unexpired portion of a lease has to be valued during which the working of shorts is permitted, and if the lessor has up to date not worked an amount of mineral corresponding to the full dead rent, there is standing to his credit an amount of prepaid royalty against which he is entitled to work mineral without the payment of further royalty. It cannot be denied that this constitutes a contingent asset, and it is a question for the valuer what the probability may be of conditions arising that will allow a quantity of mineral to be worked in excess of that corresponding to the dead rent, and what value shall be put upon this privilege. Each case must require individual consideration having regard to all the attendant circumstances, and no general rules at all can be given; it can only be said that the privilege is generally worth something, and must not be overlooked in the valuation.

The rate of royalty paid in respect of various minerals necessarily fluctuates within wide limits. It is usually fixed either at a definite figure per ton of mineral extracted, or else at a fraction of the selling price of the mineral, being often subject to a sliding scale under which this fraction itself is increased when the price of the mineral goes up. Neither method is free from objections. In the former, the mineral proprietor receives always the same amount per ton irrespective of whether the value of his mineral declines or appreciates. Under the latter, the mineral proprietor receives

an increased royalty when the price of the mineral goes up, although this may be not because its intrinsic value is any greater, but because the wages of the miners producing it have risen, which is manifestly inequitable to the lessee of the mine. A third method of payment has sometimes been adopted—namely, to make the royalty a proportion of the profits earned by the working of the mine; this again is objectionable, because it makes the mineral proprietor's royalty depend upon the skill with which the lessee works the mine, and might conceivably give the former some cause to interfere in the administration of the mine. Hence in practice the first two methods, or variations of them, are practically the only ones met with, in spite of the defects above pointed out.

In this country the royalties on coal range from about 3d. to about 2s. per ton, and probably average 7d. to 8d. at the present time; when the royalty is stated per foot per acre, the royalties are usually between £15 and £50, with an average of about £30 to £35; obviously, although differently stated, this is a tonnage royalty; taking the figure often assumed of 1,200 tons per foot per acre, a royalty of £30 per foot per acre is equal to 6d. per ton.

The royalties on oil shale in Scotland are between 3d. and 1s. per ton. The royalties on fireclay are from 3d. to 6d. per ton, and on ganister 4d. to 1s. per ton.

Royalties on Coal Measure ironstone vary from 6d. to 2s. 6d. per ton, those on Black-band being generally higher than on Clay-band ironstone.

In the Cleveland district the royalties on ironstone are usually between 4d. and 6d. per ton, in Northamptonshire 3d. to 6d., in North Lincolnshire 7d. to 1s. 6d., in Oxfordshire 2d. to 4d., the last three being usually charged by the long ton of 21 cwts.

Royalties on the red hæmatite of Cumberland and Lancashire, based on pre-war leases, run between one-sixth and one-twelfth of the selling price; owing to the immense increase in the price of this ore—practically the only Bessemer ore in the United Kingdom—during the war, royalties rose to a quite unexpected height, and in most cases a revision of these terms is now being considered. In some recent leases it has been settled at 1s. per ton when the ore sells at 15s. per ton plus 20 per cent. of any increase above that selling price.

In the Forest of Dean royalties on brown hæmatite range from 3d. to 6d. per ton.

In the Lake Superior region the average royalty on iron ore is about 30 c. per ton.*

The average royalties on igneous rocks, such as whinstone, granite, etc., usually work out at 2d. to 6d. per ton, and that on chalk and limestone is about the same or a little lower.

Royalties on slate are usually between one-twelfth and one-twentieth of the value of the dressed slates.

Royalties on the more important metalliferous minerals, such as the ores of tin, copper, lead, and zinc, are usually based on the values of the dressed minerals; on tin or copper ores the royalties range from about one-twentieth to about one-fortieth, though exceptionally as low a figure as one-eightieth has been accepted. On lead and zinc ores royalties usually range between one-twelfth and one-thirtieth, exceptionally as low as one-fortieth. These are often on a sliding scale varying with the price of the mineral or of the metal extracted therefrom; the former is the more equitable method, because the cost of the metal may vary independently of the ore, as, for instance, with the price of the coal used for

* E. E. White, *Trans. Amer. Inst. Min. Eng.* (1913), vol. xlv., p. 306.

smelting it. The following is a typical example of such a sliding scale recently settled:—

When the price of lead ore is under £15,		the royalty to be	5	per cent.
,,	,,	between £15 and £20 ,,	6	,,
,,	,,	,, £20 and £25 ,,	7	,,
,,	,,	above £25 ,,	$7\frac{1}{2}$	,,

Royalties are often levied on certain non-metallic minerals occurring in veins, the most important being fluorspar, barytes, and witherite ; the royalties on these are respectively on fluorspar 6d. to 1s. 6d., averaging about 1s. ; on barytes 9d. to 2s., averaging about 1s. 6d. ; and on witherite 1s. to 2s. 6d.

The royalty on gypsum is generally between 6d. and 1s., and on rocksalt between 2d. and 4d. per ton.

Reference must here be made to mineral wayleaves ; strictly speaking, the right to charge a wayleave depends on the ownership of land, not on the ownership of minerals. If an adit for the working of a mine passes through land owned by a landlord who has no ownership at all in the minerals in question, he is nevertheless entitled to charge wayleave on all the minerals brought through his land. In the same way a landlord can charge tunnel rent, shaft rent, water leave, etc. It might well be held to be doubtful whether a wayleave right for minerals falls properly speaking within the province of a mineral valuation, and perhaps the principal grounds for so including it depend upon the fact that the Finance (1909-10) Act of 1910 levies the due known as mineral rights duty on the rental value of all mineral wayleaves as well as on all rights to work minerals, and thus puts in this respect both classes of property upon the same footing. The expression "mineral wayleave" is fully defined in the Act (Section 24) as meaning "every wayleave, air

leave, water leave, or right to use a shaft granted to or enjoyed by the working lessee, whether above or underground for the purpose of access to or the conveyance of mineral or the ventilation or the drainage of his mine or otherwise in connection with the working of minerals," and it was further decided in Shawe Storey *v.* In. Rev. (1914, 1 K.B. 87) that the mineral rights duty is payable in respect of wayleaves irrespective of the ownership of the minerals themselves. Necessarily, where wayleaves have to be paid, they form a portion of the cost of getting the minerals, and must, therefore, be taken into account in valuing the minerals subject to them. Wayleave rents vary very greatly according to the minerals wrought and the circumstances of the case ; in respect of ironstone and coal, wayleaves in this country vary from ¼d. to 3d. per ton. According to the Royalties Commission, 1899, the average of wayleaves upon the coal worked in this country, all of which does not pay wayleaves, was then about ¼d. per ton ; to-day it is probably about ½d. per ton.

Royalties charged in other countries, even in those where minerals are held under concession, are not very different to what they are in this country. On the iron ores mined in Spain and on the Mediterranean seaboard they generally range from 6d. to 2s., and generally average about 1s. per ton. In the United States, the U.S. census returns for 1909 show that royalties on bituminous coal average about 3¼ cents per ton, but this necessarily includes a great deal of coal not worked under lease. According to Mr. G. Otis Smith,* " Royalties on bituminous coal in the United States range from about 3 to the equivalent of 35 cents per ton. A comparison of royalties paid in the United

* *U.S. Geol. Survey Bulletin* 537, " The Classification of the Public Lands " (1913), p. 80.

States shows that 10 cents a ton is not far from the average royalty paid under private leases—somewhat less in the East and somewhat more in the West ;" and Mr. H. M. Chance * states that " in the high-grade steam coal districts along the Eastern edge of the Alleghany Mountains in Pennsylvania and Maryland coal generally leased at 6, 8 and 10 cents thirty years ago has frequently been released or sub-let at royalties of 12, 15, 18 or 20 cents per ton, and such later leases made at the higher figures have often been sold for a larger bonus per acre." An anthracite royalty is even higher ; according to Mr. R. V. Norris,† " A large percentage of the anthracite coal is owned in fee by operators who lease tracts contiguous to their fee holdings. As all report royalties on the basis of tonnage produced the general average 15·5 c. per T. reported is misleading. The actual average royalty reported by operators mining generally from leased lands was 33·25 c., and by those generally mining from fee lands 5·5 c. As relatively few operators mine exclusively from either class of land, no data is available to show the actual average royalties paid, but it is believed that the present average would be approximately 40 c. per ton."

The methods adopted by the U. S. Government in valuing their coal lands for sale are highly instructive ; they are to be found in the *Bulletin* 537 and the paper by Mr. Chance above referred to. An Act of Congress of March 3rd, 1873, provided for the sale of coal lands belonging to the United States, and in carrying out this Act it was found necessary first to define what are coal lands, and, secondly, to decide what value to put upon them ; the definition was based upon the determination

* " Valuation of Coal Lands." *Trans. Amer. Inst. Min. Eng.*, vol. xlvii. (1913), p. 111.

† *Trans. Amer. Inst. Min. and Met. Eng.*, vol. lxi. (1919), p. 325.

of conditions under which a coal seam was economically workable, and, in arriving at this, three characteristics were considered—namely, the thermal value of the coal, the thickness of the seam, and its depth. It was first of all decided that 8,000 B.Th.U. shall be considered to be the minimum calorific value of a saleable and, therefore, workable coal. The minimum mineable thickness of any coal seam was taken as 14 inches for all coals having calorific values of 12,000 B.Th.U. or over; for coals of lower calorific value the minimum mineable thickness had to be increased by 1 inch for each decrease of 1,000 B.Th.U. down to 11,000 B.Th.U., by 1 inch for each 500 B.Th.U. down to 10,500 B.Th.U., by 1 inch for each 250 B.Th.U. down to 10,000 B.Th.U., and by 1 inch for each 100 B.Th.U. below this, so that the minimum mineable thickness of coal of 8,000 B.Th.U. was taken to be 38 inches. The maximum workable depth for a seam 6 feet or more was taken to be not more than 5,000 feet; for a seam of minimum mineable thickness not more than 500 feet, and for beds of any thickness between the minimum and 6 feet a depth directly proportional to that thickness. The regulation is not particularly lucid, but Mr. Chance gives it in an algebraical form as follows:—

Let D be the maximum workable depth; let t be the thickness of the seam in inches, let T be the minimum mineable thickness in inches computed as above from the calorific power, and let c be the calorific power of the coal in B.Th.U. A bed of coal 6 feet or more thick, having a calorific power of 12,000 B.Th.U. or over, is considered workable at a depth of $\frac{12,000}{3} = 4,000$ feet.

In other cases $D = \frac{t - T}{72 - T}\left(\frac{c}{3} - 500\right) + 500.$

Lands containing coal seams that comply with these conditions are classified as Coal Lands.

The purchase price of such coal is considered to be 1 cent per ton for coal having a calorific value of 12,500 B.Th.U., and apparently the quantity of recoverable coal is taken at 100 tons per inch per acre; hence, keeping to Mr. Chance's formula, the value of the coal lands in dollars per acre is given by the following expression, where d is the actual depth of the coal seam (if less than 500 feet, it is taken as 500):—

$$\left(\frac{D-d}{D-500}\right)\frac{c}{12{,}500}\left[\frac{4t}{12}+\left(\frac{t}{12}\right)^2\right].$$

It must be borne in mind that this expression gives the purchase price of the unsevered coal, and, therefore, takes into account that considerable time must elapse before the money paid for such coal can be recovered from the sale thereof; furthermore, every effort is made to make the conditions attractive to the purchaser and to get the coal lands developed.

In the zinc mining districts of Wisconsin, according to Mr. W. L. Uglow,* "The prevailing custom is for an operating company to lease ore-bearing lands from the fee owner; in consideration of this lease the company proceeds to drill, sink shafts on, and to develop and mine any ore body that is found therein. A royalty, usually 10 per cent. of the gross receipts (but sometimes varying between 5 per cent. and 15 per cent.) is paid to the fee owner as rent or his share in the earnings of the mine."

In the Commonwealth of Australia the several constituent States have adopted methods that do not vary

* "A Study of Methods of Mine Valuation and Assessment with Special Reference to the Zinc Mines of South-Western Wisconsin" (1914), p. 11.

greatly from each other in principle. Powers are given to prospect, and mining leases are granted ; in the latter a royalty, usually in the form of a percentage, is charged on gold and most other minerals except coal, upon which a royalty ranging usually from 3d. to 6d. per ton is charged ; in some cases there are also taxes to be paid in addition, and these may occasionally be quite heavy. As an example, the regulations governing mineral leases in the State of New South Wales may be quoted, these being determined by the Mining Act, No. 49, of 1906, with its subsequent amendments, the most important of which are contained in the Mining (Amendment) Act, No. 41, of 1918. These leases may be granted either on Crown lands or on private lands, the period in both cases being, as a rule, 20 years, with certain powers of renewal for a further like period. The areas granted are as follows :—

For gold dredging leases,	not exceeding	100 acres.
For gold mining leases,	,,	25 ,,
For opal mining leases,	,,	¼ acre.
For coal, oil shale, etc.,	,,	640 acres.
For other minerals,	,,	80 ,,

On Crown lands the yearly rental payable is 2s. per acre, except on dredging leases, where it is 2s. 6d. ; on private lands a rental of 20s. per acre is reserved to the owner of the land. These rentals constitute a merging dead rent. A royalty on the minerals wrought is payable to the Crown, this being, in respect of leases granted since 1918, an amount equal to 1 per cent. of the value of the gold or other minerals won, whether under mining or under dredging leases, provided that no royalty is payable if such value falls below £500 per annum, save in the case of coal or oil shale, when the royalty is 6d. per ton ; since January, 1921, however, on the renewal of leases, lessees are given the option of paying either

a flat rate royalty of 9d. per ton or a sliding scale royalty based on the selling price of the coal, as follows :—

When the selling price of the coal is under 10s. per ton the royalty shall be 6d.
" " " between 10s. and 13s. per ton the royalty shall be 7d.
" " " between 13s. and 15s. per ton the royalty shall be 8d.
" " " above 15s. per ton the royalty shall be 9d.

Furthermore, under Regulation 117 B, made in July, 1921, in the case of renewals of leases for a further term after the first renewal, the royalties above stated are replaced by royalties based upon profits instead of being based upon the value of the mineral won (except in respect of coal or shale mines), on the following scale :— When the net annual profits do not exceed £100,000 the royalty shall be 3 per cent. of such profits ; when they are between £100,000 and £200,000 the royalty shall be 3 per cent. of such profits plus a further 1 per cent. of the excess over £100,000 ; when the annual profits exceed the sum of £200,000 the royalty shall be 3 per cent. of such profits plus a further 1 per cent. on the second £100,000, and plus a further 1 per cent. on the third £100,000 or part thereof, and thereafter in the increasing proportion of 1 per cent. on each successive £100,000 or part thereof of the balance of such profits. All the royalties above named apply to minerals reserved to the Crown whether on Crown lands or on private lands, but when the minerals are owned by the owner of the land the royalty is 5 per cent. of the net annual profits, except on coal or oil shale on which the royalty is 6d. per ton. This one example will suffice to show the general trend of legislation under which mining leases in Australia are granted.

In British Columbia and Nova Scotia coal is taxed at the rate of 10 c. and 12 c. per ton respectively. In Canada royalties for the most part take the form of taxation on net profits; for instance, in Ontario the rate of taxation on profits is 3 per cent. In South Africa practically all royalties are levied in the form of taxation; thus gold mines are taxed 10 per cent. on their profits.

In valuing mines as distinct from mineral properties, the annual income is the profit resulting from the mining operations—that is to say, the difference between the price realised by the sale of the mineral and the total costs of production. These latter comprise the wages paid to all the miners, underground workers, surface workers, dressers or men engaged in preparation of the mineral for the market, the cost of stores and supplies of all kinds, management and administration, transport of the mineral to the place of sale and selling charges; if the mineral is leased the royalty rent must be included, as also must all wayleaves, rates and taxes; several foreign countries charge also an export tax upon minerals. The costs must include, not only the money actually spent in getting the mineral, but also that spent upon exploration and development work required to maintain a steady reserve of mineral ready to be worked. As a general rule, all development work until regular exploitation is started is charged to capital, whilst all such work subsequent to the general extraction of mineral forms an item in the working costs; it is by no means easy in every case to draw the line equitably between these two methods of charging costs, and there is still a good deal of difference among authorities upon the matter. It may fairly be conceded that in opening up, say, a mineral vein, the cost of sinking shafts and driving cross-cuts to intersect the vein should rank as capital outlay, and furthermore, that drivages upon the vein,

winzes in the vein, etc., until regular stoping commences, should be carried to the same account. The prolongation of these drives, the putting up of rises, etc., for the purpose of blocking out certain quantities of ore to be worked subsequently will, however, be looked upon as a development charge. The best practice appears to be to debit all such charges to a special development account, which will be credited with the value of any ore got during development work; it will further be credited for each accounting period with a sum, based on the tonnage of ore extracted, which will fairly represent the development work, the charge per ton being arrived at by dividing the total development costs in such period by the tonnage of ore thereby developed. On the other hand, when the shafts have to be sunk to a deeper level and cross-cuts driven out to intersect the vein, or if cross-cuts are driven from the vein that was being originally worked in order to open up a parallel vein, these operations distinctly increase the available assets of the mine, and are, therefore, usually charged to capital account.

Another item that has given rise and still gives rise to much discussion is that of depreciation of the plant. Apart from the cost of repairs and renewals which necessarily form a part of the working costs, it would seem right to include also the depreciation of the plant due to wear and tear, which diminishes its value, even though the plant may still be capable of doing good work and may not need immediate replacement; now-a-days, moreover, when improvements in mining methods and appliances are fortunately rapid, obsolescence must also be taken into account. There are various methods of charging depreciation. The methods to be applied in valuation must be those which will correctly represent the amount to be charged under this head, and not the

methods employed for Income Tax purposes, which cannot be considered as coming under this definition. The income tax method is based upon the calculation of depreciation upon the diminishing value of the plant —that is to say, it is charged each year upon the value of the plant less the amount of depreciation written off in the previous year. There are two grave objections to this method. In the first place, the value can never come down to zero, yet anyone, who has seen abandoned mines in out-of-the-way places and noticed the derelict machinery left there for the reason that its removal would cost more than its sale would realise in the market, must be aware that the value of mining plant does at times come down to nothing. Another, and probably a more serious objection, is that under this method the amount written off for depreciation is a maximum when the machinery is new, and becomes very small as the machinery gets older, this being exactly the opposite to what actually occurs in practice, seeing that in the first few years of its life machinery will probably depreciate very little, whilst towards the end of its career it depreciates rapidly, and it is then that obsolescence makes itself felt. Of the various methods available, perhaps the best is to estimate at the outset the probable life of the machinery, and then to write off each year an amount such that it, together with interest upon it, shall amount to the figure obtained by dividing the first cost by the life of the machines, so that the sums of all these amounts together with interest shall at the end of the term amount to the first cost.* This method may be spoken of as the sinking fund method, and may be expressed algebraically as

* See the Author's evidence before the Royal Commission for Income Tax for 1919—Third Instalment of the Minutes of Evidence, page 422 and Appendix 21.

follows:—Let P be the first cost of the plant, and n be the life of the plant in years; let p be the value of the plant at the end of the term, and let a be the amount to be written off annually. Let the rate of interest per unit be i, this being the effective rate for accumulations after the deduction of income tax.

Then for the 1st year $a = \left(\frac{P - p}{n}\right) \frac{1}{(1 + i)^{n-1}}$.

,, 2nd ,, $a = \left(\frac{P - p}{n}\right) \frac{1}{(1 + i)^{n-2}}$.

,, 3rd ,, $a = \left(\frac{P - p}{n}\right) \frac{1}{(1 + i)^{n-3}}$.

,, nth ,, $a = \frac{P - p}{n}$.

This method has the advantage that the amounts to be set aside increase as the plant grows older.

There is considerable divergence of view among engineers as to the proper rates of depreciation to be allowed owing to the different principles adopted, and in any case these rates are bound to vary within wide limits with the attention and money devoted to upkeep. As a reasonably safe rate to cover depreciation from all causes, the following may be taken as representing fairly the working lives to be allowed for the various classes of machinery—that is to say, the periods within which it may be assumed that the value falls from first cost down to scrap prices, or alternatively to zero, if the situation of the plant is such that the cost of realisation is likely to exceed such scrap value:—

Boilers,	15 to 20 years.
Engines and ordinary machinery,	8 to 15 ,,
Pumps, pipes, and heavy machines,	15 to 25 ,,
Dressing machinery,	5 to 15 ,,
Electric plant,	8 to 15 ,,
Buildings and surface structures,	20 to 40 ,,

If the exploitant leases the mine, the royalties payable to the mineral owner as well as any wayleaves form an addition to the working costs. If the mine is the property of those working it, this item does not appear so that the profit is to this extent greater; on the other hand, the cost of acquiring the mineral rights will have increased proportionately the capital outlay of the concern, so that increased revenue is required to provide an adequate return upon the increased capital as well as for amortisation. It must not be forgotten that local rates now form a serious item in the cost of working mines in this country. Unfortunately the rating of mines varies very greatly in different parts of the Kingdom, and is by no means based upon generally accepted principles, so that each case will need investigation (see p. 253).

In coal mining, the following table shows the fluctuations during the last two centuries in the proportions of the various items that enter into price of coal at the pit's mouth :—

	1700-50	1866-96	1901.	1913.	1920.
	Per cent.	Per cent.	Per cent.	Per cent.	Per cent.
Royalty, . .	12·5	8·5	7·96	5·35	1·98
Wages, . . .	37·5	66·0	67·37	62·55	72·04
Materials, . .	25·0	17·0	16·54	16·45	19·56
Administration, .				7·00	0·92
Interest and profit,	25·0	8·5	8·13	8·65	5·50

The United States Geological Survey * has attempted to collect data to show how the cost of mining coal in the United States is affected by the thickness of the

* *The U.S. Geological Survey Bulletin* 537, "The Classification of Public Lands" (1913), p. 84.

seams, and gives the following table, in which the cost of mining a seam 10 feet thick is taken as unity :—

Thickness of Seam.	Relative Cost of Mining Coal.
10 feet	1·00
9 ,,	1·02
8 ,,	1·05
7 ,,	1·10
6 ,,	1·20
5 ,,	1·33
4 ,,	1·50
3 ,,	2·00
2 ,,	3·00

It will be seen that the relative cost is approximately given by the expression—

$$\frac{100}{100 - (10 - t)^2},$$

where t is the thickness of the seam in feet. It may be doubted whether this formula would hold good for this country ; most engineers would probably look upon a seam of between 5 feet and 6 feet thick as the most economical to work, and would hold that the cost would rise as the thickness of the seam departs further from this dimension, more slowly for seams thicker than 6 feet and more rapidly for seams thinner than 5 feet, if it be assumed that all other conditions remain about the same. It is practically impossible to say anything definite as to the cost of mining in a given deposit ; this can only be estimated by a mining engineer having the suitable experience, and here, too, local knowledge is very important.

The other element besides the cost of production that enters into the calculation of profit is the estimated

value of the product ; this is often at least as difficult to forecast as the former. Before the war the values of most of the leading minerals were settling into more or less steady grooves, but to-day all values are uncertain, and there are few who would care to estimate what any mineral may be worth in a few years' time. Under the circumstances only a few indications can be given.

The price of coal had been a steadily rising one until the war upset all values ; to-day it is still fluctuating widely, and it will, of course, always fluctuate with the state of the trade and the world's demands, but it is beginning to show some tendency to remain somewhere between 20s. and 30s. per ton. It seems clear that if it were to rise as high as the latter figure, so many of the industries that are the chief users of coal would be unable to work at remunerative rates, and hence would have to close down, that the demand for coal would decrease so rapidly as to bring its price down. Again, with at any rate three countries—namely, India, China, and South Africa—possessing enormous coal reserves, and able to produce coal at one-third to one-fourth of the above minimum, it is difficult to see how high prices can be maintained in the future when these countries shall have developed their coal resources and are prepared to compete with us in the world's markets. On the other hand, it does not seem possible to-day for the British coal miner to earn a reasonable living wage with coal much below the above minimum, and since wages constitute fully two-thirds of the pit-head value of coal, the price of labour has a preponderating influence. It need hardly be said that it is the value of the ordinary grades of industrial coal that is here referred to ; special qualities, such as Welsh steam coal, will always command higher prices, and the relative

market values of the different categories necessarily fluctuate from time to time, but may be fairly well averaged over a term of years. In some countries, and notably in the United States, the method of valuing coal by its calorific power has come into extended use, and there is much to be said for the principle. The coal user, who is, after all, the final arbiter who decides what coal is worth, buys coal, not because he wants so many tons of black material, but because he wants a certain quantity of potential calorific energy. It has been already indicated that the U.S. Geological Survey takes account of the calorific value of the coal in fixing the value of coal-bearing lands, and the U.S. Government * purchases much of its coal supplies on a calorific basis, comparing the various tenders submitted upon the basis of the cost of 1,000,000 B.Th.U. under each of them.

The price of metalliferous minerals is always calculated upon the basis of the expected yield of metal therefrom. For most ores, the tonnage of metal is calculated from the assay of the ore less an assumed loss in smelting; this is multiplied by the market value of the metal, the cost of the smelting operation or the "returning charge" is deducted, and the difference is the net value of the ore at the smelting works. The value of the ore at the mine or at the point at which it is delivered to a common carrier—rail or ship—is necessarily this value less the cost of transport. Brokerage and other marketing charges must also at times be taken into account. Before the war these methods had been reduced to fairly definite formulas which had obtained a considerable measure of general acceptance. Thus copper ores were often bought and sold on the

* *U.S. Geol. Survey Bulletin* 428, "The Purchase of Coal by the Government under Specification," by George S. Pope (1910).

Rio Tinto formula. The weight paid for was the dry weight—*i.e.*, the weight as received less the amount of moisture contained therein and less an allowance of 1 per cent. for draftage. The richness of the ore was estimated by wet assay with specified deductions for smelting losses, these being about 1 per cent. for a 3 per cent. ore, and varying with the percentage of copper present. The value of the unit or of each 1 per cent. of copper per ton was 0·01 of the price of best selected copper at the date of sale less a returning charge of 2s. 3d. to 2s. 6d. per unit. The product of the net weight, the percentage of copper allowed as above, and the price of copper per unit was the value of the parcel of ore. The methods in use to-day are somewhat similar, the returning charges, however, being approximately double. Owing to its leading position to-day as a copper producer, the United States is to a large extent able to control copper prices. Much useful information upon the methods of ore valuation current in America is to be found in a paper on the subject by Mr. L. C. Stuckey.*

For tin ores the methods are somewhat similar; before the war the returning charge was about £7 10s. per ton of ore when tin was at £150; the returning charge to-day is about double this figure.

For lead ore the weight of ore paid for is the net dry weight; the percentage of lead is that obtained by dry assay in iron crucibles less 4 units—*i.e.*, 4 per cent. on the ore—for ordinary concentrates. The returning charge was usually about £2 per ton. Silver above 3 ozs. per ton was paid for at assay value at the current rate for fine silver, no payment being made for less than 3 ozs. to the ton of ore. To-day the returning charge is about £4 per ton.

* *Inst. Min. and Met. Bulletin* 210, March, 1922, "Notes on the Valuation of Ores, Concentrates and Smelter Products."

Zinc ore was usually valued by a Belgian formula, Belgium having been one of the most important purchasers of zinc ores. The weight of ore paid for was the net dry weight; the percentage of zinc allowed was the wet assay less 8 to 10, usually 9 per cent., the price allowed was 95 per cent. of the current market price for spelter, and the returning charge was about £3 per ton. To-day the returning charge would appear to be about £6 per ton.

Silver and gold ores are rarely sold as such. The silver in complex ores is usually valued as indicated above under lead ores, the cost of smelting it being covered by the returning charge for the base metal. Gold is usually produced as bullion at the mine itself, the cost of its extraction being looked upon as part of the cost of mining operations; it is sold by assay at the current market price for gold.

Iron and manganese ores are usually dealt with on a different basis. There are various categories of iron ore, the classification of which depends essentially upon the phosphorus contents. Bessemer ore is usually taken to be ore in which the phosphorus does not exceed 0·07 per cent. of the iron present. It is usually quoted at per ton c.i.f. for Rubio, which is understood to be for a standard ore containing 50 per cent. of iron, with a scale that used to be about 6d. per unit for each 1 per cent. of iron above or below that standard; the standard for silica was 8 per cent., and the ore was usually penalised 1½d. for each 1 per cent. above that figure. Especially pure ores or ores in a specially good physical condition—that is to say, very lumpy—can command prices somewhat higher than the quoted Rubio figure, whilst ores inferior in these respects can only be disposed of at prices lower than this figure. Ores suitable for making common iron with phosphorus above the Bessemer

limit, but too low for the basic Bessemer process, say under about 4 per cent. of the iron present, were about 5s. per ton below the price of Bessemer ore, whilst high phosphorus ores suitable for basic Bessemer commanded prices nearly up to those paid for Bessemer ores. In the present uncertain state of the iron and steel market it is difficult to give precise figures, but it can be stated generally that somewhat similar scales to the above are in force to-day. On to-day's prices 7½d. per unit of iron and 2½d. per unit of silica would be about equitable allowances.

The price of manganese ore depends essentially upon its chemical constituents ; ores containing a large proportion of manganese binoxide, known in the trade as "peroxide ores," and of a high standard of purity, are used for chemical purposes and for dry batteries ; the value of these ores depends upon the amount of available oxygen present—that is to say, on the amount of binoxide, the ores sold for these purposes ranging from about 80 to about 87 per cent. of binoxide ; the price is largely influenced by the amount of iron present. Ordinary ores with, say, 2 to 3 per cent. of iron are worth from £5 to £8 per ton, according to the demand ; ores containing 0·5 per cent. of iron or less are worth a considerable premium on the above figure. Other ores, used for metallurgical purposes, are valued according to the percentage of manganese present, but must be fairly low in phosphorus and silica. The price is usually between 9d. and 1s. 4d. per unit of manganese ; phosphorus in excess of 0·1 per cent. is usually penalised, say, about 4d. to 6d. per ton for each 0·01 per cent. of phosphorus in excess. Silica above 8 per cent. is penalised about 3d. per ton for each 1 per cent. above that figure.

Iron pyrites is sold on the basis of its sulphur con-

tents; the price ranges from 5d. to 8d. per unit of sulphur per ton, so that a pyrites assaying 48 per cent. of sulphur would sell for 28s. per ton when the unit of sulphur is worth 7d. Pyrites free from arsenic commands an additional premium; if the pyrites is cupriferous the copper is paid for as in copper ores.

All the above data can only be looked upon as giving general indications of the methods of valuing minerals; the prices themselves are necessarily liable to wide fluctuations in accordance with the demands of the world's markets. It is an easy matter to obtain the current quotations for the more important metals, and these, as has been seen, form the basis upon which ores are bought and sold. The mineral valuer has the much more difficult task of deciding, not what his ore is worth to-day, but what its average value is likely to be during the life of the mineral property, and it need hardly be said that this is a task that demands the highest qualities of commercial judgment and foresight.

The rate of interest that is to be expected upon the capital invested in a mineral venture must depend, as already indicated, upon the risk attaching to that venture, and is always greater than that paid upon gilt-edged securities. The rate of interest that can be obtained from time to time upon such securities may be looked upon as the rate when the risk is practically nil, or when the probability of the payment of regular interest and of the absolute security of the capital invested is practically unity. It is true that the values of even the soundest Government securities fluctuate, and that an investor in such might be compelled to sell out at a moment when the security is depreciated, so that he would thus lose a portion of his capital, but these fluctuations, generally speaking, are as much in one direction as in the other, and thus compensate each

other over a period of years, so that there really is an average stability, which is all that can be expected. The interest that can be earned by such a first-class security is, therefore, taken as the datum line, and the rate of interest to be expected in a mining venture must be equal to this rate multiplied by a factor which represents the degree of risk attaching to the venture, or is the inverse of the probability that the venture will be a success, and which may be spoken of as the "risk factor" or "risk ratio." The gilt-edged rate, sometimes spoken of as the accumulative rate, multiplied by the risk factor gives the rate of interest that money invested in the speculative enterprise ought to earn, and may be called the risk rate. Some writers speak of it as the remunerative rate, as indicating that it represents remuneration for the risk run, but the term risk rate will here be used in order to emphasise the fact that it is the ordinary rate multiplied by the risk factor, in order to compensate for the speculative nature of the enterprise. As has been previously indicated, the risk factor ought to be so selected that if a capitalist invested money in a very large number of such ventures, he would, taking the successes with the failures, realise the same interest upon his whole investments as though he had invested in gilt-edged securites; strictly speaking, this position could only arise if it were possible to invest in an infinite number of such speculative investments, but as the number is not only finite, but necessarily limited, the speculator will usually demand a somewhat higher risk factor than would be the strict mathematical equivalent of the risk run. The determination of the risk factor is perhaps the most difficult problem that the valuer has to solve. It might be quite low in such transactions, as the purchase of mineral royalties leased to a responsible lessee with the mines

in full operation, and where the probable quantity of mineral existing has been so determined as to leave but a small probable error; in such a case a factor as low as 1·5 might be accepted. The factor would reach its maximum in the case of a concern proposing to work a mineral vein in a new district, where little more than the outcrop of the vein is known, and the quantity, richness, and character of the mineral are all practically little more than conjectural. Under such conditions a risk factor of at least 6 might be required in order to attract the necessary capital. Generally speaking, risk factors seem to range between 2 and 5, the former for well-proved bedded deposits, the latter for imperfectly explored veins or masses. Necessarily, in determining the risk rate the valuer must have regard not only to the risk factor, but also to the probable interest that will be paid on gilt-edged securities during the life of the mineral property.

The risk factor is a simple enough conception, and its application as a figure by which the normal or gilt-edged rate of interest must be multiplied, presents no difficulty if its object—namely, that of compensating an investor for the risk he runs in a speculative investment—is kept in mind, the normal or gilt-edged rate being looked upon merely as a rental paid for the use of capital. A decided difficulty arises, where a high rate of income tax has to be taken into account. It has already been shown that money must be assumed to accumulate at a rate of interest i free from income tax, which is known when i_t, the ordinarily quoted gilt-edged rate including income tax and the rate of income tax are known. For example, if an investor puts £10,000 into a mining venture and expects a 10 per cent. return upon his investment, he expects to receive £1,000 per annum, upon which he would have to pay income tax,

say at 5s. in the £. In this case it does not matter to him whether the mine pays the tax and hands him £750, or whether he receives the £1,000 in full and pays out £250 to the tax collector; he could in either case consider it an investment returning him a net 7½ per cent. When, however, redemption has to be taken into account also, the question assumes a more complicated form. It need hardly be pointed out that the complication is due to the injustice which the investor in mines in this country suffers, because the income tax authorities do not allow for the redemption of the money invested in a mine, although the latter is admittedly a wasting asset.* The most convenient way appears to be to treat the transaction in the way in which it actually takes place in practice—that is to say, to look upon the mine as paying its dividends less tax and allowing the investor to calculate his return as 10 per cent. less income tax—*i.e.*, 7½ per cent. upon his investment—because the income tax has already been paid, and to bear in mind at the same time that a reserve fund of any kind can only accumulate at the net rate of interest—*i.e.*, after allowing for the deduction of income tax.

Another element in the valuation of mineral property is the first cost of bringing it into value—that is to say, of opening up and developing it, acquiring and erecting the necessary appliances, machinery and plant, in some cases including plant for cleaning, dressing, or otherwise making marketable the product, and in some cases providing means of transport and building workmen's houses, etc.; occasionally the acquisition of land, water rights, etc., has also to be undertaken. All these various items make up the capital outlay that must be

* See "The Incidence of Taxation upon Metalliferous Mining in the British Isles," by the Author. *Trans. Inst. Min. and Met.*, vol. xxvii., 1918, p. 203.

incurred before any profit can be derived from the operations of mining. Broadly speaking, of two mineral properties producing an equal revenue, that one must be the more valuable which requires the least capital outlay upon it, and the difference between the amount of capital outlay in the two cases is necessarily a measure of the difference in the respective values of the properties.

Here, again, it is impossible to lay down any general rules; seeing that all the conditions of mining and of producing a marketable product vary so widely, it is only natural that the capital required in different cases must fluctuate within very wide limits indeed. There is obviously no comparison possible between the deposit lying above water level that can be opened up by a short cross-cut and the deposit lying deep within the earth's crust and overlain by heavily water-logged strata or beds of quicksand. Some properties, like gold-bearing alluvials, have been opened up by a digger with no capital beyond that required for a shovel and a pan; in other cases preliminary equipment has cost hundreds of thousands of pounds. Some minerals, like certain iron ores, can be dug out by steam shovel and loaded direct into railway waggons for transport to the purchaser, whereas others, like certain tin ores and gold ores, need a complex and elaborate system of crushing and dressing operations supplemented by a number of chemical processes, for the whole of which a large and costly equipment is required. The only attempt at generalisation has been made with respect to coal mines, where there is in many cases sufficient general similarity to enable averages to be arrived at; in this country it was generally considered in pre-war days that a colliery would cost on the average about 10s. for each ton of coal raised per annum, though it must be under-

stood that this was an average figure, and that exceptionally the capital cost might fall below 5s., and in other cases might exceed 15s. To-day the corresponding average figure would probably lie between 20s. and 30s., due not only to the increase in the cost of labour and materials, but largely to the fact that the greater part of the coal that remains to be won in this country lies deep, and is in many cases overlain by strata carrying large quantities of water, which adds immensely to the cost of sinking. Thus a number of figures of sinking costs in Westphalia, published in 1901 by L. Hoffmann,* showed that, whereas a relatively dry sinking through shales to a depth of about 1,300 feet cost from £30 to £55 per yard, in a similar wet shaft the cost would rise to £125, and that costs would increase still more rapidly in proportion as the sinkings attained greater depths. Mr. H. M. Chance † gives the following data as to the first cost of coal mining in America under pre-war conditions :—

"When the coal to be worked is of good thickness, 5 feet or more, is nearly level, with sufficient grade towards the outcrop for good drainage, is soft and easily cut, lies from 25 to 40 feet above the railroad shipping tracks, is in best demand as run-of-mines coal, is clean and requires no preparation other than care in mining, the plant and improvements will be of the cheapest and most simple type and the investment in plant and mine development may not exceed 50 cents per ton of annual capacity, or, say, $50,000 to $60,000, for a capacity of 120,000 tons (240 days at 500 tons) annually.

* "Leistungen und Kosten beim Shachtabteufen im Ruhrbezirk," *Glückauf*, vol. xxxvii. (1910), p. 775.

† "Valuation of Coal Land." *Trans. Amer. Inst. Min. Eng.*, vol. xlvii. (1913), p. 133.

"If the coal contains slate, requiring hand-picking, if its best market is as screened coal sold in several sizes, if it lies high above the railroad, if it is hard and not easily cut, if it is not self-draining and is not nearly level, the plant and improvements and development work may require an investment of perhaps $1 per ton of annual capacity. The difference is due to the extra cost of plane and equipment, to additional cost of shipping tracks, to cost of tipple with shaking screens and picking table, to larger power plant, more mining machines, greater cost of driving entries and other mine development work, to expense caused by drainage troubles, etc.

"If the coal is relatively thin, requiring more entry work at higher cost per foot, if it lies below water level and must be opened by shaft or slope, if the slack and nut require jigging, if the property lies some distance from a railroad and a branch road must be built to reach it, the capital required may easily be $1.50, and in some cases may reach $2 per ton of annual output."

The modern tendency in all mining work, owing to the increasing cost of labour, is to replace manual operations wherever possible by mechanical appliances; the rock drill, which has practically everywhere taken the place of the old hammer and borer, is a typical example; it is only necessary to refer to such appliances as coal-cutters, face conveyors, and the underground loaders, which are being introduced in the United States, to show the prevalence of this tendency. This means that efforts are being made to cut down working costs at the expense of increased outlay for plant, and this tendency is likely to be even more accentuated in the future, owing to the fact that modern mining development is tending more and more to the treatment of low-grade deposits on a very large scale, a proposition that would be economically impossible unless working costs

were cut down to a minimum by the extensive employment of mechanical means for handling very large quantities of material. Capital outlay is, therefore, likely to become continuously greater in the future, and due regard must be had to this point in estimating the capital required for any proposed mining undertaking. It is generally admitted that the expenditure of £1,000 on plant is warranted if the labour of one man can be saved thereby. A prudent engineer, in estimating his capital requirements, will take into account, not only the immediate needs of the property which he is called upon to develop, but will also endeavour to foresee how costs may be reduced by the introduction of mechanical appliances, and should endeavour to provide for these in his capital estimates.

The composition of the mineral deposit, upon which its value depends, is determined by sampling; this operation, which is a laborious one, needing expenditure of considerable time and money, is in many cases one of the most important in the process of mineral valuation. In general terms sampling consists in breaking or cutting out of the mineral deposit certain portions so taken as to represent as accurately as possible the average composition of the mineral deposit at the point of sampling. The sample so taken is broken down and reduced in bulk until an aliquot part is obtained, by methods which shall cause it to represent as accurately as possible the average composition of the total sample. In this form, the sample which then usually weighs only a few pounds, is submitted to assay, using this word in its widest possible significance; the assayer operates on a small portion of the sample usually varying from a few grains to an ounce or two. It will thus be seen that ultimately the composition is determined by some form of chemical analysis of a very small quantity

taken as representing the composition of the entire sample, which may weigh up to several hundredweights, and the composition of all the samples thus ascertained is made use of in order to arrive at the average composition of the mineral deposit, which may contain some hundreds of thousands of tons. Here, therefore, the applicability of the assay result to the final valuation depends essentially upon the laws of probability, and the same remarks are applicable to this portion of the work as to the determination of quantity. In this connection it must be borne in mind that the value of a very large class of mineral deposits is determined by the tonnage (depending upon the area multiplied by the thickness) multiplied by the assay value per ton. If the thickness of a deposit be T units and the assay value V units per ton, its value will be a function of the product V T; it is, however, clear that neither the thickness nor the assay value can be determined with absolute accuracy, and they should, therefore be written $T \pm t$ and $V \pm v$ respectively, where t and v are the respective probable errors. The probable error of the product V T is $\sqrt{T^2 v^2 + V^2 t^2}$, so that the value of the deposit is a function of $V T \pm \sqrt{T^2 v^2 + V^2 t^2}$. It is convenient to express t and v as percentages of T and V respectively; let the corresponding percentages be a per cent. and b per cent., so that

$$a = \frac{100\,t}{T}, \text{ and } b = \frac{100\,v}{V}.$$

Then the probable error of the product

$$\sqrt{T^2 v^2 + V^2 t^2} = \sqrt{T^2 \frac{b^2 V^2}{100^2} + V^2 \frac{a^2 T^2}{100^2}} = \frac{T V}{100}\sqrt{b^2 + a^2},$$

or the probable error of the product is $\sqrt{a^2 + b^2}$ per cent. of the product T V.

It may be taken for granted that assays, usually done in duplicate, so as to exclude accidental errors, are practically accurate within the limits required for the work in hand. It is, however, obvious that the small amount of mineral operated on by the assayer does not with certainty represent the average of the samples submitted to him any more than this sample represents with certainty the average of the entire quantity broken out in the first instance. All that can be said is that by taking care that, before taking an aliquot part of the sample, the latter consists of a sufficiently large number of fragments of approximately equal size, the probability that the aliquot part will correctly represent the average of the sample may be made as great as may be desired by increasing the number and proportionately decreasing the size of the particles. Even at best it is only a probability and never a certainty. The same laws of probability which indicate that the great majority of such samples will closely approximate to the average also indicate that exceptionally a sample must be produced which diverges considerably from that average. In taking the samples in the first instance exactly the same position arises ; for instance, in sampling a vein which has been opened up by means of a drift upon it, it would be in accordance with sound practice to sample this vein by taking cuts across it at regular intervals, say 10 feet apart, each of these cuts being supposed to represent the average composition of the mineral for 5 feet on either side of the cut. It is, of course, theoretically possible that every one of these cuts might be made in exceptionally rich or exceptionally poor ore, and that the cut would not in the least represent the composition of the intervening portion of the ground. The larger the number of these cuts the greater the probability that their average will accurately represent the

average of the entire deposit, but at the very best it is only a probability and never a certainty, and this fact cannot be too emphatically insisted upon.

The methods of taking the sample vary to some extent with the nature of the deposit to be examined and with facilities available for obtaining samples. Thus in the case of a bed, say of ironstone, the thickness and character might be proved entirely by boreholes; in such a case the cores produced in boring may with advantage be split lengthways, and the half of the core thus obtained may constitute the sample. Care must be taken that the length of core thus sampled is equal to the thickness as measured for the deposit. If the deposit has been opened by trial pits, each pit may be sampled by cutting a vertical groove from roof to floor of the deposit, say 6 inches wide by 2 inches deep, and collecting the whole of the material thus cut out; it is best to cut two such grooves on opposite sides of the shaft; according to the nature of the deposit, that is its apparent homogeneity or otherwise, the contents of the two grooves may be united to make one sample, or they may be assayed separately. Before attempting to cut the sample groove, the face of the deposit should be carefully cleaned or scraped down; if the shaft is an old one and the deposit weathered, it is a good plan to cut a groove, say an inch deep, in the first instance, the whole of the material produced by which is rejected, and then to deepen the shallow groove thus cut to the required depth in order to obtain the sample. If a bed has been opened out by levels or workings in it, samples may in a similar way be cut from roof to floor at suitable points in the workings, the same proportions being maintained. If possible, samples should be taken in such a way as to be distributed with reasonable regularity over the area within which the work can be done,

and a sample plan should be prepared showing the exact spot at which each sample was taken. The samples should be numbered, not necessarily consecutively, and numbers inserted on the plan to correspond to the numbers on the sample bags. The practice of different engineers varies considerably in detail. The writer's practice is to have bags of stout, closely woven canvas, with numbers painted clearly on the outside in waterproof paint. If the samples have to undergo an exceptionally long and hard journey before reaching an assay office, it is as well also to put inside each sample bag a small tin disc with the same number stamped on; the tin disc should be attached by a thin cord to the inside of the bag so that there is no risk of a disc with one number being accidentally put into a bag bearing another number. The sample bags should be tied up with good cord, and the author's practice is always to seal them with a lead seal bearing his own private marks, which can easily be changed from time to time. Sometimes, especially in sampling large mines of precious metals, the inspecting engineer will make his own assays on the spot, or will have an assayer with him for the same purpose. More generally, however, the samples are sent to an assayer, and it is advisable that the latter should be able to certify that he received the samples with the seals intact; it is, as a rule, right that the assayer's certificate should be appended to the engineer's valuation.

If the mineral bed is seen to include layers which will be rejected in the course of mining, these should be kept out of the samples, their widths being, of course, carefully recorded. Thus, take the case of a coal seam having in its middle a band of shale 6 inches thick: the whole of this would be rejected either in the course of mining the coal or on the picking belts before the

coal is sent to market, and it would, therefore, be right that this shale should be excluded in sampling the coal seam, the thickness of the latter being, therefore, taken as being 6 inches less than the thickness from roof to floor. If, however, instead of one band of shale, there were, say, a dozen narrow seams, each of which does not exceed half an inch in thickness, so that it would be quite impossible to keep them out in the course of mining, they should be included in the sample. If it were necessary that the dirt should be got rid of by crushing and washing, as might be the case if the coal were intended for coke-making, the cost of this treatment would have to be allowed for as an item in the cost of production, but the sample should represent the coal as it would be sent to the washery. The guiding principle must always be that the sample of a mineral deposit must be taken in the same way that the deposit itself will be gotten in the subsequent mining operations, and should, therefore, as nearly as possible represent the quality of mineral that these mining operations would produce. Many beds of ironstone are naturally divided into layers more or less regular, the richness of which may differ considerably from each other. In such a case it is advisable to sample and assay the various sections separately; it may easily happen that a band may be so siliceous or so low in iron that its inclusion might seriously detract from the value of the entire deposit, whilst its exclusion, though decreasing the amount of available ore, might so raise its market value as to make the latter a more profitable way of dealing with the deposit; in such a case the valuer should naturally sample the bed in the same way as it will ultimately be mined. Thus in many parts of the Northamptonshire ore field the lowest portion, say about 2 feet, of the deposit is so siliceous as not to be

marketable, and, as a general rule, this is left behind when the ore is worked; although in geological fact a portion of the deposit, economically it should not be considered as forming part of it. It need hardly be said that local experience is invaluable in such work, since it often requires an eye practised in discriminating between small differences in appearance of that particular deposit to tell where the line as between one layer and the next should be drawn. The sample of ironstone should weigh about 10 lbs. for each foot in thickness of the bed, the sample of a coal seam about half that amount. These samples are generally best cut with a small sharp pick; this is always sufficient for sampling coal, and generally for ironstone also; if the latter is very hard a gad and hand hammer may have to be used. The chippings produced are best caught in a box, though for the last foot or so down to the floor a sack laid upon the ground close up to the face that is being sampled is preferable.

Sampling the auriferous deposits on the Witwatersrand has become a highly developed system, this sampling being specifically not merely for the purposes of valuation, but in order to guide the management in their work of exploitation. The principles are, however, the same.* The auriferous reef, usually in several sections, is exposed in the levels, either in the roof or side; at intervals, usually regular intervals, a sample is cut from wall to wall across each section of reef exposed, the width of the reef being at the same time recorded. Samples are usually taken at 10-foot intervals, but,

* See "The Witwatersrand Gold Fields Banket and Mining Practice," by S. J. Truscott, p. 305; "The Sampling and Estimation of Ore in a Mine," by T. A. Rickard, p. 131; "Continuous Section System of Mine Sampling," by M. H. Burnham, *Trans. Inst. Min. Met.*, vol. x. (1901-2), p. 204; "Modern Mine Valuation," by M. H. Burnham (1912).

where assays show considerable variations, 5-foot intervals are sometimes adopted. It is, of course, not always possible to take such samples at absolutely regular intervals; when the intervals are irregular the intervals between the samples are carefully measured. The exact position of the samples is marked on an assay plan on which the width and the assay are also recorded. It is convenient to make the return in the form of dwts. of gold per ton per inch of width, the inch-dwt. or some similar composite figure showing the relative total gold contents of the reef. This only needs multiplying by the length to which the sample applies and by a factor which is readily calculated in order to give the total gold contents per unit of depth. If it be assumed that the vein-stuff runs, for example, 15 cubic feet to the ton, the number of inch-dwts. multiplied by the average width in inches and by the length to which the sample applies in feet and divided by 180 will give the number of dwts. of gold contained in a block of the reef of the given length and 1 foot in depth. The generally accepted method of calculating the assay value is as follows:—Let W be the width of the reef at any sampling point and R its richness, and let w and r be the width and richness respectively at the next sampling point, the distance between the two being l. It is assumed that over the distance l the width varies uniformly from W to w, and the gold contents also uniformly from R to r; the number of inch-dwts. are given by the expression $\left(\frac{\mathrm{W\,R} + w\,r}{2}\right)$, and this divided by $\frac{\mathrm{W} + w}{2}$, or $\frac{\mathrm{W\,R} + w\,r}{\mathrm{W} + w}$ is the average assay value in dwts. per ton. A repetition of this calculation over a series of samples and of measurements will give the total gold contents over the entire length sampled

] er unit of depth ; in sampling along a drift, for example, it is quite usual to take samples and measure widths every 10 feet, and to calculate accordingly.

This is the accepted Witwatersrand practice, and it apparently gives results near enough to be acceptable. It can, however, readily be shown that it is not accurate. Let Fig. 11 represent the plan of a block of unit depth, and of length l, the width in inches and assay value of the sample at one end being W and R respectively, and at the other end w and r. Consider any narrow strip of length $d\,x$ at distance x from the end $r\,w$; its width is $w + \frac{x}{l}(\mathrm{W} - w)$, and its richness is $r + \frac{x}{l}(\mathrm{R} - r)$.

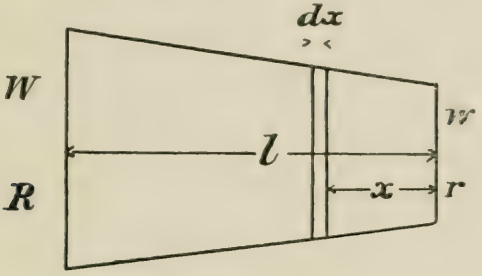


Fig. 11.

Hence its value in inch-dwts. is

$$\left(w + \frac{x}{l}(\mathrm{W} - w)\right) d\,x \left(r + \frac{x}{l}(\mathrm{R} - r)\right).$$

The total value of the entire area

$$= \int_0^l \left(w + \frac{\mathrm{W} - w}{l} x\right)\left(r + \frac{\mathrm{R} - r}{l} x\right) d\,x$$

$$= \int_0 \left(w\,r + \frac{w(\mathrm{R}-r) + r(\mathrm{W}-w)}{l} x + \frac{(\mathrm{W}-w)(\mathrm{R}-r)}{l^2} x^2\right) d\,x$$

$$= \left(w\,r\,x + \frac{w(\mathrm{R}-r) + r(\mathrm{W}-w)}{l} \frac{x^2}{2} + \frac{(\mathrm{W}-w)(\mathrm{R}-r)}{l^2} \frac{x^3}{3}\right)_0^l$$

$$= w\,r\,l + \frac{(w\,\mathrm{R} - w\,r + \mathrm{W}\,r - w\,r)l}{2} + \frac{(\mathrm{W}\,\mathrm{R} - \mathrm{W}r - w\,\mathrm{R} + w\,r)l}{3}$$

or, when x becomes ultimately equal to l,

$$= \frac{l}{6}(6\,w\,r + 3\,w\,\mathrm{R} - 3\,w\,r + 3\,\mathrm{W}\,r - 3\,w\,r + 2\,\mathrm{W}\,\mathrm{R} - 2\,\mathrm{W}\,r - 2\,w\,\mathrm{R} + 2\,w\,r)$$

$$= \frac{l}{6}(2\,w\,r + w\,\mathrm{R} + \mathrm{W}\,r + 2\,\mathrm{W}\,\mathrm{R}).$$

The area of the block $= \frac{\mathrm{W} + w}{2}\,l$. Hence the average assay value

$$= \frac{\frac{l}{6}(2\,w\,r + w\,\mathrm{R} + \mathrm{W}\,r + 2\,\mathrm{W}\,\mathrm{R})}{\frac{l}{2}(\mathrm{W} + w)}$$

$$= \frac{2\,w\,r + 2\,\mathrm{W}\,\mathrm{R} + w\,\mathrm{R} + \mathrm{W}\,r}{3\,(\mathrm{W} + w)}$$

$$= \frac{3\,(w\,r + \mathrm{W}\,\mathrm{R}) + (w\,\mathrm{R} + \mathrm{W}\,r - w\,r - \mathrm{W}\,\mathrm{R})}{3\,(\mathrm{W} + w)}$$

$$= \frac{w\,r + \mathrm{W}\,\mathrm{R}}{\mathrm{W} + w} + \frac{(w - \mathrm{W})\,(\mathrm{R} - r)}{3\,(\mathrm{W} + w)}.$$

This is the same result as that obtained by Mr. Ross Hoffman*; it is, however, obtained in a somewhat more direct manner, for which the author is indebted to his colleague, Mr. J. W. Bullerwell, M.Sc. It will be seen that the true assay value differs from the accepted value by $\frac{(w - \mathrm{W})\,(\mathrm{R} - r)}{3\,(\mathrm{W} + w)}$, which becomes small when the assays do not differ greatly from each other, and when the widths also do not differ greatly, and are at the same time large; the expression only becomes zero, when either the widths or the assay values are the same at both ends; it becomes negative when both

* "The Sampling and Estimation of Ore in a Mine," by T. A. Rickard, p. 35.

assay value and richness increase or decrease in the same direction, and positive when the increment or decrement is in opposite directions in the block under consideration. When a series of assays are thus taken along a level, $\frac{(w - W)(R - r)}{3(W + w)}$ is likely in the long run to be as often positive as negative, and, as it is generally small, the resultant error may well become negligible. This, however, is far from being the case in each individual calculation. Thus Truscott * gives the following series of determinations at three points, A, B, and C :—

A,	10	inches and	10	dwts.	= 100	inches	×	dwts.
B,	12	,,	20	,,	= 240	,,	×	,,
C,	8	,,	5	,,	= 40	,,	×	,,

He gives the average thickness between A and B as 11 inches, and the average assay value as 15·45 dwts., following the accepted Witwatersrand method, and similarly for B and C 10 inches and 14·00 dwts. ; calculations by the correct formula given above make the average values 14·15 dwts. and 13 dwts. respectively, so that the incorrect method in use involves an error of 7 per cent. in the latter case. It may be noted that the expression

$$\frac{w\,r + W\,R}{W + w} + \frac{(w - W)(R - r)}{3(W + w)}$$

may also be written

$$\frac{w\,r + W\,R}{3(W + w)} + \frac{R + r}{3},$$

which is rather more convenient for calculation.

The question may fairly be asked whether the Witwatersrand method of thus taking the width into account

* *Op. cit.*, p. 319.

when averaging assay values should or should not be generally adopted in preference to the method above given of averaging values without regard to widths. The answer must depend upon the conditions of the case. In the first place, the Witwatersrand method is only applicable to deposits reasonably well opened, where samples are taken fairly close together; furthermore, it postulates that both richness and width shall vary regularly according to a straight-line law from sample cut to sample cut; it is evident that if in Fig. 11 the ore body pinched or widened considerably between the two cuts at either end, the whole calculation would be thrown out, and the same thing would happen were the middle portion of the block much richer or much poorer than the samples. The variations in widths can readily enough be seen, and Mr. M. H. Burnham proposes to get over the difficulty due to such variations by cutting his samples wherever the ore body commences to either narrow or widen, or, in his words,* "where there is a marked variation in thickness, the sounder procedure is to space the samples with reference to such changes, especially where preliminary assays indicate great variation in value, afterwards taking intermediate sample sections." Even this precaution, however, will not avail where the assay values vary irregularly and capriciously. It would appear to follow that under ordinary conditions, where levels or other workings are not available for samples to be taken fairly close together, or where there is evidence of great and irregular variability in richness, the previously given method of averaging without regard to width is probably preferable, unless it is clear from an inspection of the results that width of vein and richness have some well-marked connection with

* *Op. cit.*, p. 113.

each other, as, for instance, if it be found that the narrowest portions of a deposit tend to be definitely richer or definitely poorer than the wider portions.

In the case of mineral veins, there is no better method of taking the samples than the one above indicated as used on the Witwatersrand; a groove must be cut across the vein from wall to wall and all the material cut out must be collected. The cut is best done by hammer and moil, a two-handed hammer being necessary in the case of hard quartz. Where compressed air is available, a small jack hammer, hand-held, fitted with steels with a chisel edge can render excellent service in this method of sampling. At the United Eastern Mining Company's Mines, Oatman, Ariz.,* light machines are used with $\frac{3}{4}$-inch moils, cutting channels 2 inches wide 1 inch deep at 5 ft. intervals, the cuttings being caught on a sheet of heavy canvas 8 ft. by 10 ft. If the vein appears to show rich and poor streaks, these should be sampled separately, the widths of each being measured with very great care.

Theoretically, the groove ought to be cut across the vein from wall to wall at right angles to these, but this is not always possible. As long as the groove runs straight from wall to wall—that is to say, makes everywhere the same angle with the dip of the deposit—a correct sample will be obtained. The writer prefers a horizontal groove under all circumstances, and measures the horizontal width along the line on which the sample is cut. Such a groove can be cut along the side of a shaft, along the roof of a level, or across the face. It sometimes happens, however, that it becomes necessary to cut samples in a level, the roof of which is markedly

* "Cutting Mine Samples with Machines," Donald C. Gilbert. *Engineering and Mining Journal-Press*, Aug. 26, 1922, p. 375.

arched, and in such a case it is impossible to cut a horizontal groove. Thus, let *a b c d* (Fig. 12) be the section of such an arched drift in a vein as shown. The portion *b c* is practically horizontal, whereas the portion *c d* approaches more nearly to the vertical, and it can at once be seen that, whilst these two lengths correspond approximately to equal widths of vein, the length of vein-stuff exposed along *c d* is roughly three times as great as that in *b c*. If a uniform groove were cut right round the arch there would be, say, three

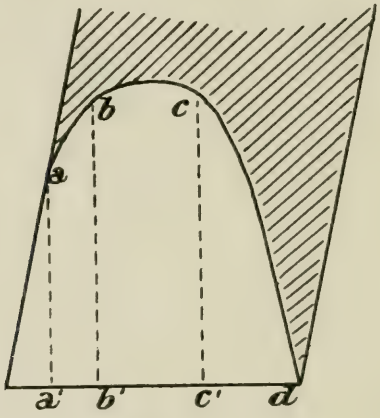


Fig. 12.

times as much material representing the portion of the vein corresponding to *c d* as that corresponding to *b c*, and, if the foot-wall portion were markedly richer or poorer than the rest of the vein, the sample would be seriously inaccurate; several methods may be used for overcoming the difficulty. The writer would sample separately each of the sections *a b*, *b c*, and *c d*; by dropping perpendiculars from the points *a*, *b*, *c* on to a horizontal line at *a′ b′ c′*, the horizontal line *a d* is divided proportionately to the true widths of the vein between the points *a* and *b*, *b* and *c*, and *c* and *d*; *a′ b′*, *b′ c′*, and *c′ d* are measured, and

portions of the three samples are weighed off so that the weights shall be in the ratio of the widths so measured. The three lots are then mixed and form a true representative sample of the vein.

Occasionally, on account of excessive hardness or irregularity, it is almost impossible to chip a groove in the manner indicated across the vein; in that case other devices have to be resorted to. One method is to place a row of pop-shots across the vein from wall to wall, lay a platform of planks or sacking on the floor of the level, and shoot the vein down upon it; if the shots are quite short, an accurate sample may be obtained in this way, but care must be taken to see that the vein is broken out regularly. Some engineers are entirely opposed to the blasting out of samples, and argue with considerable force that any vein that can be drilled to take a shot hole can be broken out by the use of the moil. It is quite evident that a vein carrying precious metals, irregularly distributed, requires far closer and more accurate sampling than a vein of base metal showing considerable uniformity; in the same way, if the difference between the pay-shoots and the intervening low-grade vein-stuff is readily distinguished by the eye, samples may be placed further apart in the latter than in the former. In some cases samples have been taken with as little interval between them as 2 feet. In the porphyry copper mines of the Miami district, "all drifts, cross-cuts, and raises within the general ore zone are sampled at intervals of 5 feet, and the assay results are entered on detailed assay maps." * As regards the size of the sample, it appears safe practice to follow the figures given by Mr. Philip Argall †:—"In gold and silver mines from 2 to 10 lbs.

* *Op. cit.*, p. 92.

† "The Sampling and Estimation of Ore in a Mine," by T. A. Rickard, p. 103.

should cover the limits in ordinary cases, and an average of, say, 5 lbs. per foot will usually be found quite safe. Thus a vein 10 feet wide would give a 50-lb. sample. This factor determined for the particular vein under consideration, the samplers should be instructed as to the width and depths of the sample cuts required, and, as a check on the accuracy of their work, the sample should be weighed, and those exceeding 10 per cent. variation from the adopted standard rejected." Mr. M. H. Burnham * " stipulates for not less than 20 lbs. weight per running foot for sampling gold ores and more if the ground be ' blocky.' "

In the case of irregular deposits sampling becomes much more difficult; it is rare that the deposit in its earlier stages is exposed from wall to wall, and in the majority of cases the valuer will have to content himself with sampling such portions of the deposit as are accessible; often he may have to rely entirely upon drill cores. In a very wide face, cutting a groove is not always necessary, and pieces may be broken off at intervals. A stout moil and two-handed sledge hammer, or, when available, a jack hammer drill with chisel edge, or small pop shots may be used for breaking off pieces of 1 to 2 lbs. in weight, taking care that all the pieces are of practically equal size, from spots spaced at quite uniform distances apart across the deposit from wall to wall if accessible, whenever exceptionally wide deposits have to be dealt with. This method can be employed quite safely, and is perhaps to be preferred to any other when sampling across masses of relatively low grade and comparatively uniform quality, such as are met with in certain deposits of iron ore, iron pyrites, etc. Here again each case must be considered by itself, and no general rules can possibly be

* *Op. cit.*, p. 111.

laid down; experience alone can help the engineer to decide what number of samples he should take and how far they should be spaced apart to enable him to arrive with some confidence at the probable average value of the deposit. It must not be forgotten that in some masses, particularly perhaps in the case of lenticular deposits, there is at times a liability for the composition to vary considerably as between the external and the central parts of the mass. A case in point is the Chillagoe Mine, Queensland, worked on a lenticular body of cupriferous pyrites, the external portions of which were very much richer in copper than the interior; the former naturally were the first to be cut and sampled, and the inspecting engineers, who had come to the conclusion that these samples represented the average composition of the mine, were thus seriously led astray. This branch of the work is perhaps the most important part of the field work connected with valuation. Before leaving it, it may be well to refer briefly to the possibilities which it offers to fraud on the part of the vendor of a mining property. Whilst the risk of such "salting" may not be very great in the case of a coal or an ironstone property, a gold mining or diamond mining proposition offers obvious possibilities. Every experienced mining engineer is familiar with various devices of this kind, such as shooting gold-dust with a gun into the faces of a soft deposit, supplying the inspecting engineer's party with dynamite containing gold-dust, injecting gold chloride solution with a syringe into the bag containing the samples, substituting rich quartz for poor, arranging to drop pieces of rich ore into the bucket whilst it is being hoisted up the shaft, etc. Even in the case of base metal mines, particularly copper and nickel, similar devices have been resorted to. The best advice that can be given to the valuing engineer is

invariably to go through precisely the same routine in taking his samples, whether he suspects the possibility of fraud or whether he is convinced that anything of the kind is out of the question. He or a confidential assistant should never allow the sample out of sight from the time that it is being taken in the mine until it has been broken down and the proper portion sealed in the bag for transmission to the assayer, whilst the sample bags should always remain under the care of the engineer or of the trusted assistant. It is also a good plan to take some of the samples in duplicate.

After the sample has been obtained, it must be reduced in bulk by taking an aliquot representative part thereof; in the absence of machinery, this is usually done by repeated halving. Occasionally, though rarely, it happens that the sampler has at his command suitable machinery for breaking down the sample together with mechanical sampling appliances. There are a number of mechanical samplers on the market, the most satisfactory being those that deflect the entire stream of crushed ore at definite intervals, and thus take out a determined fraction thereof. More often the valuing engineer has no such appliances; in some cases he can take with him a hand-power rockbreaker for breaking down his samples, which is a great advantage, but more often even the transport of such an appliance offers insuperable difficulties. The breaking must then be done by hand. A sheet of stout canvas laid on a platform of boards or on a well-levelled piece of ground should be used, and, if possible, surrounded by a frame of boards on edge, 6 inches to 9 inches deep. The breaking may be done on old stamp-dies or on any old castings or other masses of iron or steel that may be available; for example, anvils and even heavy sledge hammers may

be used in default of anything better, and in extreme cases, boulders of a tough fine-grained rock, preferably of a basic igneous rock, may be used, the breaking being usually done with hammers about 2 lbs. in weight. When the sample has been broken down it must be thoroughly mixed and halved; the method usually adopted is that of coning and quartering, and, if carefully done, this method is quite reliable; it requires, however, a little knack to make a proper cone, each shovelful being discharged by a turn of the wrist, so as to drop on to the apex of the cone and to roll uniformly all down the sides. The cone is then carefully flattened and divided into four equal quarters; two opposite quarters are rejected, any pieces in the other half that are too large are broken further, and the operation repeated as far as may be desired. Instead of quartering, devices like the split shovel or a set of riffles, or the improved riffles, known as the Jones Sampler, may be used. The principle of all of these is to alternately collect and reject strips of crushed ore of equal width. A particularly convenient sampling device, which is less known in this country than it deserves to be, consists of three troughs of wood or metal, V-shaped in cross-section, with the apical angle about 90°, and the edge forming the apex sharp. The size depends upon the size of the particles. For a sample in which no piece is larger than about $\frac{1}{2}$ inch, the troughs can be from 2 to 3 feet long with sides 3 inches to 4 inches wide, pieces of angle iron answering admirably. As shown in isometric projection in the sub-joined diagram (Fig. 13), one piece of angle iron is laid edge upwards, best over a couple of boxes, as there indicated. Another piece of troughing is placed edge downwards so that its edge rests on the apex of the lower piece near the middle of its length, it being inclined at such an angle

that the ore will slide down freely. A third piece is placed also edge downwards, so as to discharge near the top of the second piece, sloping in the opposite direction to the second piece. Care must be taken that the edges of the three pieces of troughing are all in the same vertical plane. The crushed ore is thrown

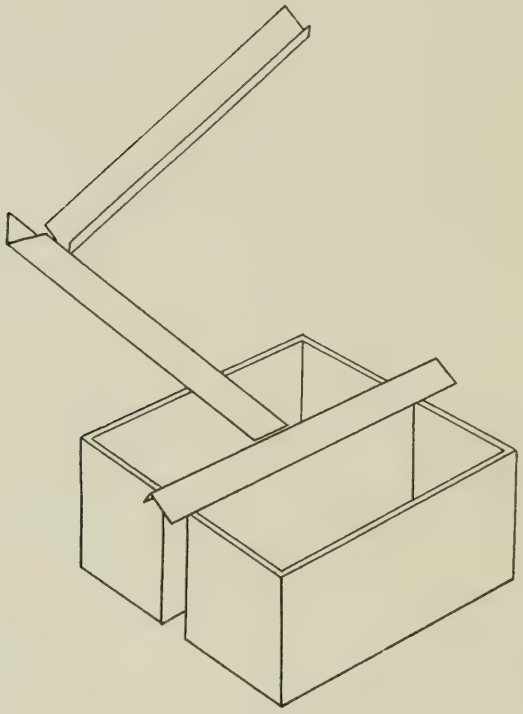


Fig. 13.

or dropped into the uppermost trough ; sliding down this, it strikes the second, down which it slides, and as the pieces strike the edge of the lower inverted piece of troughing they drop indiscriminately right or left, so that the sample is thus divided into two equal parts. The correctness of the sampling may be judged by weighing each half, when there should not be more than

5 per cent. difference between them. One-half is rejected and the other further reduced. It is highly advisable, in all sampling, to keep carefully each rejected half in turn until the next in order is ready, so that, should any accident happen to the sample that is being prepared, the other half may be substituted for it, and in this way the work of obtaining a sample be not lost. The weight of the ultimate sample will be the 2^{-n}th part of the original sample, where n represents the number of cuts. The size to which the sample should be broken depends on the nature of the mineral, and should be smaller in the case of an ore containing high values irregularly disseminated than in the case of minerals of relatively uniform composition. As a general rule, the following may be taken as representing convenient practice :—

When weight of sample	exceeds	1,000	lbs.	no piece	should exceed	$1\frac{3}{4}$ inches.
,,	,,	,,	200	,,	,,	1 inch.
,,	,,	,,	40	,,	,,	$\frac{1}{2}$,,
,,	,,	,,	10	,,	,,	$\frac{1}{4}$,,
,,	,,	is under	10	,,	,,	$\frac{3}{16}$,,

These figures may be taken as applicable to ordinary ores and minerals ; in the case of such an ore as a rich gold ore, where the precious metal is irregularly distributed, the pieces may be somewhat smaller, whereas in the case of a comparatively homogeneous ore in which the valuable ingredient constitutes a large percentage, as in the case of iron ore or iron pyrites, they may be somewhat larger. The final sample is generally between 5 and 14 lbs.

Occasionally it happens that samples may have to be still further reduced so as to be sent by post ; in that case an iron plate and a bucking hammer or a large

iron mortar and pestle will be required, and the halving is best done by means of a special set of small riffles, a small trough sampler, such as illustrated in Fig. 13, or an appliance like Clarkson's Hand Sampler. No definite rules can be laid down, as much depends upon the accessibility of the mineral property, upon the appliances that may be found there, and the methods of transport that are available.

CHAPTER V.

FORMULAS AND CALCULATIONS.

ONCE the various data have been collected as indicated in the preceding pages, the calculation of the value of the mineral property depends upon certain mathematical considerations, the application of which presents no serious difficulties when the underlying principles are thoroughly understood. Where divergencies of opinion occur, as they not unfrequently do among different valuers, it will generally be found that these are due to the fact that they have assumed, often tacitly, that there are different conditions to be fulfilled as the basis of the valuation.

The basal principle may be said to be the ordinary compound interest formula which is a simple summation of a geometrical series as given in any text-book of elementary Algebra. If a sum of P pounds be invested at a rate of interest equal to i per annum per unit—that is to say, at one hundredth of the generally quoted rate per cent., the interest being payable at the end of each year, the capital will amount to $P + P\,i$ or $P\,(1 + i)$ at the end of the first year; this sum will amount to $P\,(1 + i)^2$ at the end of the second year, and to $P\,(1+i)^n$ at the end of n years. Conversely, a sum of money S due at the end of n years hence is worth actually $\frac{S}{(1 + i)^n}$, because this quantity will amount to S at the end of n years if allowed to accumulate at the given rate of interest.

This statement is subject to the proviso that i shall

be the rate of interest actually earned—that is to say, free from all deductions, such as taxes, rates, etc., and in this country specifically free from income tax. If the ordinarily quoted annual rate of interest per unit, including tax, is i_t, and the rate of income tax is x shillings in the pound, $i = \frac{(20 - x)}{20} i_t$; if the capital be invested in the purchase of mineral rights it becomes further liable to mineral rights duty in accordance with the Finance (1909-10) Act, 1910, S. 20, which enacts that such duty shall be levied on the rental value of all rights to work minerals and of all mineral way-leaves; it is not charged in respect of "common clay, common brick clay, common brick earth, or sand, chalk, limestone, or gravel." The duty by the above Act was fixed at one shilling in the pound, and has since remained at that rate.

When the income is derived from such ownership of mineral rights, and mineral rights duty has to be paid as well as income tax, this is chargeable on the income after deduction of the latter tax. At the present rate of mineral rights duty the net rate of interest obtainable as revenue derived from such sources is

$$\frac{19}{20}\left(\frac{20 - x}{20}\right) i_t \text{ or } \frac{380 - 19x}{400} i_t.$$

The revenue itself is, of course, diminished in the same proportion.

In the above formulas for income tax only the ordinary rate of taxation has here been considered, although the rate actually payable may often be different. For example, the income may be liable to super-tax in the case of an individual or individuals, or to the Corporation Profits Tax * in case of a limited company,

* The Finance Act, 1920 (Geo. V. 10 and 11, C. 18), Part V.

or the income may be derived from a colliery in which income tax is based on the average of the previous five years' workings, and not on the income of the year under consideration. It is, however, impossible to represent all such special cases by a general formula, and on this account only the ordinary rate has here been considered. Whenever the circumstances are accurately known, it is easy to insert in the calculation a rate of tax appropriate to them instead of the ordinary standard rate. It must be noted that income tax is payable upon the whole of the income inclusive of any portion that may be set aside to form a reserve for a sinking fund or for any other purposes. Shareholders in a company which has paid income tax in this way will receive dividend warrants which are free from income tax, inasmuch as this tax will have been already paid at source ; furthermore, it may be noted that if any portion of such reserve fund is distributed to the shareholders at any subsequent time, it will not be again subject to income tax, any sum so carried forward from profits being carried forward as a taxed reserve. In all calculations, however, involving mines owned in Britain, the incidence of income tax, etc., upon the whole of the profits must be borne in mind. There are certain special cases where companies working abroad may be taxed only on such part of the profits as are remitted to this country, but this point will be dealt with more fully in the next chapter. It follows from the above that if two mines are otherwise exactly equal in all respects save that one is situated in this country and one abroad, a higher actual value will be placed upon the mine abroad if the conditions are such that the sinking fund to replace the capital can be accumulated abroad without being liable to income tax.

Furthermore, in practice interest, dividends, royal-

ties, etc., are, as a rule, paid at shorter intervals than one year, often every six months and sometimes quarterly; in the former case n must be taken to represent twice the number of years and i as one-half of the rate of annual interest per unit, and in the second case n must be taken to represent four times the number of years and i one-fourth of the annual rate of interest per unit. Generally n is taken to represent the number of accounting periods and i the rate of interest per unit earned during such an accounting period—that is to say, the annual rate divided by the number of periods within the year.

The above statement represents what is usually intended when it is stipulated that the rate of interest shall be a certain rate per cent. per annum, and that the interest shall be paid at shorter intervals than one year. The shorter the interval the greater is the effective rate of interest actually received, because the interest paid at the end of the first accounting period is itself earning interest during the remainder of the year. When this is not the intention, but when the stipulated rate is intended to be the actual rate per annum, although it may be paid at shorter intervals than one year, it may be necessary to calculate what rate of interest payable at such shorter intervals is the effective equivalent of the given rate of interest per annum or *vice versâ*, so that the owner of the capital shall be in precisely the same position whether the annual payment of interest at one rate or the payment of interest at shorter intervals at the other rate be adopted.

Let there be x intervals or accounting periods in the year, and let the effective rate to be paid be j per unit to correspond with a rate of interest i per unit per annum.

Then $(1 + j)^n = \left(1 + \frac{i}{x}\right)^{nx}$.

Whence $1 + j = \left(1 + \frac{i}{x}\right)^x$. . . . . (1)

Occasionally it is stipulated that a sum of P pounds shall be invested to earn a given rate of interest s per unit, whilst the interest thus earned shall be allowed to accumulate at another (usually a lower) rate. The interest received for each accounting period will be P s, and if this accumulates for n periods at a rate of interest i, it (together with the original capital) will at the end of n periods amount to a sum S, such that

$$S = P + \frac{P s}{i}[(1 + i)^n - 1]. \quad . \quad . \quad (2)$$

Conversely it may be required to find the actual value P of a sum of money S due at the end of n accounting periods under the same conditions, when

$$P = \frac{S i}{i + s[(1 + i)^n - 1]}. \quad . \quad . \quad (3)$$

If a be an annuity (that is to say, a sum of money payable from any source whatever at regular intervals) payable at the end of each of n accounting periods, the rate of interest per unit during such period being i, the actual value * of such an annuity is $\frac{a}{i}\left[\frac{(1 + i)^n - 1}{(1 + i)^n}\right]$; this formula is sometimes spoken of as Inwood's formula. Values of the expression $\frac{(1 + i)^n - 1}{i(1 + i)^n}$ for different values

* The term "actual value" is here employed throughout instead of present value, which is the more usual expression, in order to avoid confusion when dealing with deferred annuities. By actual value is meant the value at the commencement of the accounting periods, by present value the value at the commencement of a period of deferment. When there is no deferment either phrase may be used indifferently.

of i and n can be obtained from any annuity tables, where the values of $(1+i)^n$ and $\frac{1}{(1+i)^n}$ will also be found.

The above formula may be arrived at in a number of different ways, several of which may here be examined.

I. The actual value of the sum a due at the end of the

$$\text{1st period} = \frac{a}{(1+i)},$$

$$\text{2nd} \quad ,, \quad = \frac{a}{(1+i)^2},$$

$$n\text{th} \quad ,, \quad = \frac{a}{(1+i)^n}.$$

If A is the actual value,

$$\begin{aligned} A &= \Sigma\left(\frac{a}{1+i} + \frac{a}{(1+i)^2} + \ldots \frac{a}{(1+i)^n}\right) \\ &= a\,\Sigma\left(\frac{1}{1+i} + \frac{1}{(1+i)^2} + \ldots \frac{1}{(1+i)^n}\right) \\ &= \frac{a}{1+i}\left(\frac{1-\left(\frac{1}{1+i}\right)^n}{1-\frac{1}{1+i}}\right) \\ &= \frac{a}{i}\cdot\frac{(1+i)^n-1}{(1+i)^n}, \text{ or } \frac{a}{i}[1-(1+i)^{-n}]. \quad . \quad (4) \end{aligned}$$

II. The amount of n annuities of a pounds each, accumulating at rate of interest i is the sum of the series

$$a(1+i)^{n-1} + a(1+i)^{n-2} \ldots + a = \frac{a}{i}[(1+i)^n - 1].$$

This ought to be equal to the amount of the actual value A at the end of n accounting periods, or

$$A(1+i)^n = \frac{a}{i}[(1+i)^n - 1].$$

Whence $$A = \frac{a}{i}\ \frac{(1+i)^n-1}{(1+i)^n}, \text{ as before.}$$

III. The annuity may be looked upon as applied in part to pay interest on the actual value, in part to the redemption or reduction of the capital, forming a sinking fund, interest being assumed to be paid only on such portion of the actual capital as remains unredeemed at the end of each accounting period.

Let the reduction of capital at the end of the

1st accounting period be x_1
2nd ,, ,, x_2
nth ,, ,, x_n.

At the end of the

1st accounting period $a = \mathrm{A}\,i + x_1$
2nd ,, ,, $a = (\mathrm{A} - x_1)\,i + x_2$
3rd ,, ,, $a = (\mathrm{A} - x_1 - x_2)\,i + x_3$
nth ,, ,, $a = i[\mathrm{A} - (x_1 + x_2 + x_3 \ldots + x_{n-1})] + x_n$.

$$\mathrm{A}\,i + x_1 = (\mathrm{A} - x_1)\,i + x_2 \text{ or } x_2 = x_1\,(1 + i)\,;$$

similarly, $x_3 = x_2\,(1 + i) = x_1\,(1 + i)^2$

and $x_n = x_1\,(1 + i)^{n-1}$.

$$\mathrm{A} = \Sigma\,(x_1 + x_2 + x_3 + \ldots x_n)$$
$$= x_1\,\Sigma\,[1 + (1 + i) + (1 + i)^2 + \ldots (1 + i)^{n-1}]$$
$$= x_1\,\frac{(1 + i)^n - 1}{i}.$$

$$a = \mathrm{A}\,i + x_1, \text{ or } x_1 = a - \mathrm{A}\,i.$$

$$\mathrm{A} = (a - \mathrm{A}\,i)\,\frac{(1 + i)^n - 1}{i}.$$

$$\mathrm{A}\,[1 + (1 + i)^n - 1] = \frac{a}{i}\,[(1 + i)^n - 1].$$

Whence $$\mathrm{A} = \frac{a}{i}\,\frac{(1 + i)^n - 1}{(1 + i)^n}, \text{ as before.}$$

This method is useful because it indicates what portion of the annuity may be looked upon as interest on capital and what portion of it may be looked upon as devoted to redeeming the capital invested. It is

evident that the annuity must exceed the interest on the actual value for each accounting period, the difference being $a - A\,i$; inserting the value above obtained for A, this becomes

$$a - i\left(\frac{a}{i}\,\frac{(1+i)^n - 1}{(1+i)^n}\right) = a\left(1 - \frac{(1+i)^n - 1}{(1+i)^n}\right)$$

$$= \frac{a}{(1+i)^n}.$$

If this difference be allowed to accumulate at interest i for n accounting periods it amounts to

$$\frac{a}{(1+i)^n} \cdot \frac{(1+i)^n - 1}{i} = A.$$

Thus the difference between the annuity and the interest payable at the end of each accounting period, if allowed to accumulate at interest i will amount to the actual value at the end of the term. Hence the purchaser of an annuity could spend annually an amount equal to the interest on the original purchase price, and, by allowing the balance to accumulate at the same rate of interest, he would get his purchase price back at the end of the operation, and would thus be, as naturally he ought to be, in the same position as a man who invested a capital sum equal to A at interest i, and spent the whole of the interest. The above formula may, therefore, be looked upon as a true annuity formula for determining the amount of an annuity equitably corresponding to its purchase for a given sum under the conditions set forth; it accordingly answers the tests of such a formula, and upon it all the ordinary annuity tables are constructed, these tables giving the value of the expression $\frac{(1+i)^n - 1}{i\,(1+i)^n}$ for a number of different values of i (usually stated as percentages),

and a large number of different values of n, usually between 1 and 100.

The statement that the purchaser of an annuity who spends annually an amount equal to the interest on the purchase price thereof would be in the same position as a man who invested a sum of money at the same rate of interest for a period of years equal to the term of the annuity has been shown to be mathematically true. It is, however, only true if the interest is taken in the sense of net interest or interest free of income tax. Under existing conditions the method of levying income tax operates very adversely to the annuitant, because the income tax Commissioners assess him upon the whole of his annuity as though it were a true income, instead of being, as in fact it is, part income and part reimbursement of capital.

For example, if a man invests £1,000 in Government or similar securities, say, at 4 per cent. per annum, he will receive £40 a year, on which he pays income tax at the rate of, say, 5s. in the £, and has, therefore, £30 a year to spend, so that at the end of any given term, say 20 years, he will have been spending his £30 per annum and have his £1,000 intact. Equitably, he ought to be in no better and no worse position than a man who bought an annuity for 20 years at 4 per cent. for £1,000, spent as much yearly as the first man, and invested the balance so as to accumulate at the same rate of interest. Under the above conditions the sum of £1,000 will purchase an annuity of £73·583. Disregarding income tax for the moment, if the annuitant spent, like the first man, £40 yearly, and invested the balance, £33·583, each year at 4 per cent., it would amount to £1,000 in 20 years, as it obviously should do. In order to place the annuitant in the same position as the first investor, he should be called upon to pay

£10 income tax per annum on his real income—namely, the true interest on his capital—and be allowed to use the above £33·583 free of income tax to replace his original capital, because this portion of his revenue is not income derived from his investment, but the replacement of a portion of his capital. He will, however, in fact be taxed upon the whole of his annuity, and will have to pay in income tax £18·396, leaving him with £55·187; if he spends now £30 a year, like the first man, he will only have £25·187 per annum to invest, and this, at the end of the term of 20 years, would, if accumulated at 4 per cent., amount only to £750, so that one-fourth of the capital would have been paid away as income tax in this transaction. This, however, is not all; the annuitant can only accumulate his sinking fund by investing the money, and the revenue from these investments will again be taxed, so that, instead of accumulating at 4 per cent., they will in reality accumulate at 4 per cent. less income tax, or 3 per cent., and will, therefore, at the end of 20 years amount to £676·79—that is to say, a tax supposed to be levied only upon income at the rate of 25 per cent. has in 20 years deprived the annuitant of nearly one-third of his capital.

The fact that income tax operates in this way is one that the investor must not lose sight of; he must from the outset take into account that whenever he invests in a wasting asset the tax collector takes toll of his capital as well as of his income, and that at a rate even higher than the declared rate of income tax. As all mining investments, whether in mines or in mineral properties, are necessarily investments in wasting assets, the British system of income tax assessment discriminates adversely against speculation in mines, and thus directly hinders the development

of one of the most important of our national resources.*

A capital sum simply invested at a given rate of interest will continue to earn that interest for an unlimited time, and may, therefore, be regarded as the purchase price of the actual value of a perpetual annuity, usually spoken of as a perpetuity, equal to the interest in question. Some valuers calculate the actual value of long-term annuities—*e.g.*, the income derivable from a fifty-years' lease—as though they were perpetuities, by which assumption the calculation is considerably simplified, whilst the error involved might not be a very serious one when the term is long and the rate of interest high. The relation between perpetuities and annuities for different terms and at different rates of interest is shown in the following table :—

Interest per cent.	Actual Value of £100 per Annum at the given Rates of Interest of an Annuity for			
	a Perpetuity.	30 years.	40 Years.	50 Years.
1½	£8,000	£2,488·9	£3,132·7	£3,701·3
2½	4,000	2,093·0	2,510·3	2,836·2
5	2,000	1,537·2	1,715·9	1,825·6
10	1,000	942·7	977·9	991·5
15	666·7	656·6	664·2	666·1

This approximate method of substituting perpetuities is not to be recommended ; unless used with great caution it is liable to lead to error, and it obscures the cardinal fact that all mineral and mining investments

* See " The Incidence of Taxation upon Metalliferous Mining in the British Isles," by Henry Louis. *Trans. Inst. Min. Met.*, vol. xxvii. (1918), p. 203 ; also the Author's Evidence, Royal Commission on the Income Tax, Third Instalment of the Minutes of Evidence (Cmd. 288-3, 1919), p. 422.

are necessarily from their nature invariably wasting assets.

It has been seen that the actual value of an annuity of a per annum at rate of interest i for n years is $a\frac{(1+i)^n-1}{i(1+i)^n}$, and that the amount S of this annuity at the end of n years is $(1+i)^n$ times as much, or $a\frac{(1+i)^n-1}{i}$. Hence it follows that the reciprocal of this fraction or $\frac{i}{(1+i)^n-1}$ times the annuity is the sinking fund that must be set aside in order to redeem S at the end of n years, so that

$$a = \frac{Si}{(1+i)^n-1}. \quad . \quad . \quad . \quad (5)$$

It is sometimes necessary to determine the actual value of annuities which increase or decrease uniformly either in arithmetical or in geometrical progression.

Let the annuity payable at the end of the 1st accounting period be a as before, at the end of the 2nd period $a+b$, at the end of the 3rd period $a+2b$, and at the end of the nth period $a+(n-1)b$.

The actual value of the annuity due at the end of the

$$\text{1st period} = \frac{a}{1+i}.$$

$$\text{2nd ,,} = \frac{a+b}{(1+i)^2}.$$

$$\text{3rd ,,} = \frac{a+2b}{(1+i)^3}.$$

$$n\text{th ,,} = \frac{a+(n-1)b}{(1+i)^n}.$$

The actual value

$$= \Sigma \left[\frac{a}{1+i} + \frac{a+b}{(1+i)^2} + \frac{a+2b}{(1+i)^3} + \ldots \frac{a+(n-1)b}{(1+i)^n} \right]$$

$$= \Sigma_n \left[\frac{a}{1+i} + \frac{a}{(1+i)^2} \ldots \frac{a}{(1+i)^n} + \frac{b}{(1+i)^2} + \frac{2b}{(1+i)^3} + \ldots \frac{(n-1)b}{(1+i)^n} \right]$$

$$= a\left(\frac{(1+i)^n - 1}{(i\,(1+i)^n)}\right) + \frac{b}{i}\left(\frac{(1+i)^{n-1} - 1}{i\,(1+i)^{n-1}}\right) - \frac{(n-1)\,b}{i}\left(\frac{1}{(1+i)^n}\right) \quad . \quad . \quad . \quad .(6)$$

The expressions in brackets can be obtained from the ordinary tables; $\frac{(1+i)^n - 1}{i\,(1+i)^n}$ and $\frac{(1+i)^{n-1} - 1}{i\,(1+i)^{n-1}}$ are actual values of annuities of £1 at rate of interest i for n and for $n-1$ years respectively, and $\frac{1}{(1+i)^n}$ is the actual value of £1 due n years hence at rate of interest i. If the annuity is a decreasing instead of an increasing one, $+b$ is represented in the above formula by $-b$ in the usual way.

Let the annuity payable at the end of the

1st	accounting	period	be a
2nd	,,	,,	$k\,a$
3rd	,,	,,	$k^2\,a$
nth	,,	,,	$k^{n-1}\,a$.

The actual value of the annuity at the end of the

1st period $= \frac{a}{1+i}$.	3rd period $= \frac{k^2\,a}{(1+i)^3}$.
2nd ,, $= \frac{k\,a}{(1+i)^2}$.	nth ,, $= \frac{k^{n-1}\,a}{(1+i)^n}$.

The actual value

$$= \Sigma_n \left(\frac{a}{1+i} + \frac{a\,k}{(1+i)^2} + \frac{a\,k^2}{(1+i)^3} + \ldots \frac{a\,k^{n-1}}{(1+i)^n} \right)$$

$$= \frac{a}{1+i} \Sigma_n \left(1 + \frac{k}{1+i} + \frac{k^2}{(1+i)^2} + \ldots \frac{k^{n-1}}{(1+i)^{n-1}} \right)$$

$$= \frac{a}{(1+i)^n} \cdot \frac{k^n - (1+i)^n}{k - (1+i)} \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Here, again, tables afford some assistance, as $(1+i)^n$ is the sum to which one pound will amount in n years at rate of interest i. When the annuity is an increasing one, k will be greater than unity, and if it is a decreasing one k will be a fraction less than unity.

The case sometimes arises that the annuity may be payable several times in the year, generally half-yearly or quarterly, whilst the accumulative interest is payable at different intervals. It is, therefore, useful to obtain a general expression to cover all such cases.

Let an annuity a_1 per annum be payable y times in the year, so that each payment is equal to $\frac{a_1}{y}$, whilst interest at the rate i_1 per unit per annum is payable x times during the year. The rate of accumulation is, therefore, $\left(1 + \frac{i_1}{x}\right)^x$ per annum, and in $\frac{1}{y}$ of a year it is $\left(1 + \frac{i_1}{x}\right)^{\frac{x}{y}}$*. It is required to determine the actual value of the annuity a_1 for a term of n_1 years under the above conditions. The amount payable at the end of $\frac{1}{y}$ year is $\frac{a_1}{y}$ and this at the end of the period of n_1 or $\frac{n_1 y}{y}$ years will amount to $\frac{a_1}{y}\left(1 + \frac{i_1}{x}\right)^{\frac{(n_1 y - 1)x}{y}}$. The

* On this point, see "The Theory of Finance," by Mr. George King, Third Edition, p. 7.

amount $\frac{a_1}{y}$ payable at the end of $\frac{2}{y}$ year will amount at the end of n_1 years to $\frac{a_1}{y}\left(1+\frac{i_1}{x}\right)^{\frac{(n_1 y-2)x}{y}}$. The amount $\frac{a_1}{y}$ payable at the end of $\frac{3}{y}$ year will amount at the end of n_1 years to $\frac{a_1}{y}\left(1+\frac{i_1}{x}\right)^{\frac{(n_1 y-3)x}{y}}$. The amount payable at the end of $\frac{n_1 y-2}{y}$ years will at the end of n_1 years amount to $\frac{a_1}{y}\left(1+\frac{i_1}{x}\right)^{\frac{2x}{y}}$. The amount payable at the end of $\frac{n_1 y-1}{y}$ years will amount at the end of n_1 years to $\frac{a_1}{y}\left(1+\frac{i_1}{x}\right)^{\frac{x}{y}}$. The amount payable at the end of $\frac{n_1 y}{y}$ years is $\frac{a_1}{y}$.

The value of these annuities at the end of n_1 years will, therefore, be the sum of all these amounts, or

$$\Sigma\left[\frac{a_1}{y}+\frac{a_1}{y}\left(1+\frac{i_1}{x}\right)^{\frac{x}{y}}+\frac{a_1}{y}\left(1+\frac{i_1}{x}\right)^{\frac{2x}{y}}+\ldots\right.$$
$$\left.\frac{a_1}{y}\left(1+\frac{i_1}{x}\right)^{\frac{(n_1 y-2)x}{y}}+\frac{a_1}{y}\left(1+\frac{i_1}{x}\right)^{\frac{(n_1 y-1)x}{y}}\right]$$
$$=\frac{a_1}{y}\cdot\frac{\left(1+\frac{i_1}{x}\right)^{\frac{x}{y}\cdot n_1 y}-1}{\left(1+\frac{i_1}{x}\right)^{\frac{x}{y}}-1}=\frac{a_1}{y}\cdot\frac{\left(1+\frac{i_1}{x}\right)^{n_1 x}-1}{\left(1+\frac{i_1}{x}\right)^{\frac{x}{y}}-1}$$

This sum must be equal to the sum of the actual value A accumulating at interest $\frac{i_1}{x}$ for $n_1 x$ periods, or $A\left(1+\frac{i_1}{x}\right)^{n_1 x}$.

Equating:—

$$A\left(1+\frac{i_1}{x}\right)^{n_1 x} = \frac{a_1}{y} \cdot \frac{\left(1+\frac{i_1}{x}\right)^{n_1 x} - 1}{\left(1+\frac{i_1}{x}\right)^{\frac{x}{y}} - 1}.$$

Whence

$$A = \frac{a_1}{y} \cdot \frac{\left(1+\frac{i_1}{x}\right)^{n_1 x} - 1}{\left(1+\frac{i_1}{x}\right)^{n_1 x}\left[\left(1+\frac{i_1}{x}\right)^{\frac{x}{y}} - 1\right]}. \quad (8)$$

If both periods are the same or $x = y$, this expression becomes

$$\frac{a_1}{i_1} \cdot \frac{\left(1+\frac{i_1}{x}\right)^{n_1 x} - 1}{\left(1+\frac{i_1}{x}\right)^{n_1 x}}. \quad . \quad . \quad . \quad (9)$$

which, it will be seen, is the annuity formula (No. 4), when interest at the rate $\frac{i_1}{x}$ per unit is paid $n_1 x$ times during the term of the annuity.

It should be particularly noted that when this method of calculation is employed, the symbols a_1, i_1, and n_1 mean specifically annuity per annum, interest per unit per annum and number of years, and do not refer to the accounting period, as they would do if formula (4) were employed.

Formula (4) for determining the actual value is

$$A = \frac{a}{i} \cdot \frac{(1+i)^n - 1}{(1+i)^n},$$

where a and i represent the annuity and interest respectively per accounting period, and n the number of such periods. Let a_1, i_1, and n_1 refer in the same way to the whole year; if there be x accounting periods in the year $a = \frac{a_1}{x}$, $i = \frac{i_1}{x}$ and $n = n_1 x$, so that the above

expression becomes :

$$\frac{a_1/x}{i_1/x} \cdot \frac{\left(1+\frac{i_1}{x}\right)^{n_1 x}-1}{\left(1+\frac{i_1}{x}\right)^{n_1 x}},$$

which is the same as that obtained in formula (9) above. Either method of calculation may be employed, but it is most important to note carefully the meaning attaching to the symbols in either case. Generally the symbols a, i, etc., will be used when an accounting period is referred to, and a_1, i_1, etc., when a year is specifically referred to, but it must not be forgotten that in many cases the year is the accounting period. The difference in notation will only be employed when it is required to draw specific attention to the difference in formulas according as they apply to any accounting period or to the period of a full year sub-divided into shorter periods. It is hoped that in this way all confusion may be avoided, and, in any case, specific attention will always be drawn in the text to those formulas applicable to the year only as distinguished from the more general formulas where the accounting period may be a year or any shorter or even longer time.

It has already been pointed out that, owing to the hazardous nature of mining enterprises, which are based always on probabilities and never on absolutely ascertained facts, investors in mining ventures demand a higher rate of interest than that obtained on gilt-edged securities, and that this latter rate must be multiplied by a risk factor r in order to obtain the higher rate of interest expected from mining investments. Let this risk rate of interest be s, so that $s = r\,i$. It was first pointed out by H. D. Hoskold that in such cases the ordinary annuity formula is not properly speaking applicable, because that portion of the annuity which

is assumed to accumulate in order to amount at the end of the term to the original investment is not in fact invested so as to bear interest at the risk rate, but can only be assumed to accumulate at an ordinary or gilt-edged rate. It must be borne in mind that this accumulation is in most cases a fiction ; some mining companies do indeed set aside out of their profits sums of money supposed to form a sinking fund to redeem the value of the actual investment ; the practice of setting aside such money is no doubt a good one, because emergencies arise during the careers of most mining companies, due to some mining accident, to the need for acquiring additional property or further appliances, which can be met out of such a reserve fund. For these purposes the practice of forming a reserve fund is of vital importance to mining companies, but it may fairly be questioned whether it would be legitimate to set money aside merely for the purpose of repaying the original investment when the life of the mine is terminated. It can well be imagined that an investor in mining properties would say that he would prefer to face the fact that his capital would be exhausted when the mine was worked out, and would himself make such provision for the replacement of his capital as he thought necessary, but that he would prefer to receive the whole of the profits in the form of dividends ; he might fairly argue that money set aside by a directorate to form a sinking fund must necessarily be invested in gilt-edged securities bearing a very low rate of interest, while if he had the control of this money, he could invest it himself to far better advantage, and that in this way he would be better off if he could receive the whole of his share of the profits as they accrued rather than allow them to be invested in, say, Government securities.

Whatever weight may be held to attach to these

arguments, the fact remains that, in calculating the value of an annuity derivable, *e.g.*, from mines or mineral rights, the replacement of the original investment at the end of the term of the operation must always be provided for in the calculation; otherwise comparisons between such investments and investments in ordinary securities of the non-wasting class become impossible.

If interest be allowed at the risk rate s upon the actual value or the capital invested in the mineral property, while the difference between the annuity or dividend paid in each accounting period and the interest upon the capital invested in the mineral property during such period is assumed to be set aside and to accumulate at an accumulative or gilt-edged rate of interest i, so that the total accumulations at the end of the term are equal to the actual value or original investment A, the condition of the problem as above set out will be fulfilled.

The amount available for investment at the end of each accounting period is $a - \mathrm{A}\,s$; accumulating at rate i, it will amount to $\frac{a - \mathrm{A}\,s}{i}[(1 + i)^n - 1]$, and this must amount to A at the end of n periods, so that

$$\mathrm{A} = a - \mathrm{A}\,s\,\frac{(1 + i)^n}{i}.$$

Solving for A,

$$\mathrm{A} = \frac{a[(1 + i)^n - 1]}{i + s[(1 + i)^n - 1]} = \frac{a}{s + \frac{i}{(1 + i)^n - 1}}. \qquad (10)$$

The latter form is the more convenient when a set of annuity tables is available. The expression $\frac{(1 + i)^n - 1}{n}$ has been seen to be the sum to which n annuities of £1

accumulating at interest i will amount; hence its reciprocal is the annuity which under the same conditions will amount to £1 at the end of the term, so that $\frac{i}{(1+i)^n - 1}$, usually spoken of as the sinking fund formula, will be found under that head in actuarial tables.

Inwood's method of performing the above calculation accordingly consists in taking out from the tables the sinking fund at the accumulative rate of interest for the given period which at the end of the time will amount to £1. To this is added the risk rate of interest per unit. These two amounts together, if paid periodically for the given number of accounting periods, would suffice to pay interest at the risk rate on an original loan of £1, and to replace the £1 by accumulation at the accumulative rate of interest. The actual value of an annuity equal to the above sum is, therefore, obviously unity, and the actual value of an annuity of £1 on the above terms is the reciprocal of the above sum, or the given annuity divided by the sum arrived at as above represents the actual value. This method somewhat shortens the calculation if a table of sinking funds is at hand in which the given rate of accumulative interest is included.

It need hardly be pointed out that in mineral valuations the annuity is the royalty paid on the minerals during a given accounting period if mineral rights are being valued, or the profit derivable from the mining operations if a mining proposition is the object of the valuation.

It may sometimes happen that two accounting periods of different lengths are involved; thus the annuity may be derived from an investment in a mine where dividends are payable half-yearly, whilst it may

be a condition of the accumulation of that part of the annuity that is to form the sinking fund that interest shall only be paid yearly.

As the former is an actual operation, whilst the latter is generally a hypothetical one, it is frequently assumed that the latter period coincides with the former. If it is desired to take the difference into account, one method is to reduce everything to the accounting period corresponding to the payments at risk rates, and to replace the accumulative rate of interest i by a rate j, such that a sum of money invested at this latter rate with interest payable at the end of each of the accounting periods at which the risk interest is paid, shall produce the same amount as though it had been invested at rate i, the interest being payable at the end of the accounting period corresponding with this rate. Thus, if the accumulative interest be payable yearly, and the risk interest at the end of accounting periods equal to $\frac{1}{x}$ of a year, it is necessary to substitute for rate $\frac{i}{x}$ a rate j, such that the amount of £1 at the end of the year shall be the same whether it is paid at the end of the year at rate i or whether the interest j is paid x times in the year, as in Formula (1), p. 153. Generally speaking, in such cases the difference in the accounting periods is disregarded and the accumulative rate i is allowed; the error thus caused is by no means a serious one, and the calculation is simpler.

As before, however, a general expression can be obtained for this case also.

If the actual value A is assumed to be invested at a risk rate s_1 payable y times in the year, so that the rate of interest for each period is $\frac{s_1}{y}$, whilst the accumu-

lative interest i_1 is payable x times in the year, the rate of interest for each accumulation being $\frac{i_1}{x}$, it is required to find the corresponding value of the annuity a_1 per annum for a term of n_1 years, where again the symbols a_1, i_1, s_1 and n_1 refer to yearly periods.

The amount available for investment at the end of each annuity payment is $\frac{a_1 - \mathrm{A}\, s_1}{y}$; this is available for investment y times in the year, and each time earns interest equal to $\frac{a_1 - \mathrm{A}\, s_1}{y}\left(1 + \frac{i_1}{x}\right)^{\frac{x}{y}}$ during the next $\frac{1}{x}$ of a year. The sum of all these investments, which must amount to A, is

$$\frac{a_1 - \mathrm{A}\, s_1}{y} \cdot \frac{\left(1 + \frac{i_1}{x}\right)^{n_1 x} - 1}{\left(1 + \frac{i_1}{x}\right)^{\frac{x}{y}} - 1}.$$

Equating

$$\mathrm{A} = \frac{a_1 - \mathrm{A}\, s_1}{y} \cdot \frac{\left(1 + \frac{i_1}{x}\right)^{n_1 x} - 1}{\left(1 + \frac{i_1}{x}\right)^{\frac{x}{y}} - 1}.$$

Whence

$$\mathrm{A} = \frac{a_1\left[\left(1 + \frac{i_1}{x}\right)^{n_1 x} - 1\right]}{y\left[\left(1 + \frac{i_1}{x}\right)^{\frac{x}{y}} - 1\right] + s_1\left[\left(1 + \frac{i_1}{x}\right)^{n_1 x} - 1\right]}. \qquad (11)$$

When the periods are equal or $x = y$ the above expression becomes

$$\mathrm{A} = \frac{a_1\left[\left(1 + \frac{i_1}{x}\right)^{n_1 x} - 1\right]}{i_1 + s_1\left[\left(1 + \frac{i_1}{x}\right)^{n_1 x} - 1\right]}. \qquad . \quad . \quad (12)$$

Hoskold's formula and the ordinary formula can be made to give the same result in any given case by selecting a suitable rate of risk interest, or, in other words, by suitably modifying the risk ratio. Thus, remembering that s equals $r\,i$, Hoskold's formula may be written

$$A = \frac{a}{r\,i + \dfrac{i}{(1+i)^n - 1}}.$$

Should some other risk factor, say r_0, be selected for the ordinary formula, this then becomes

$$A = a\,\frac{(1 + i\,r_0)^n - 1}{i\,r_0\,(1 + i\,r_0)^n}.$$

Equating these two values :

$$\frac{1}{r\,i + \dfrac{i}{(1+i)^n - 1}} = \frac{(1+i\,r_0)^n - 1}{i\,r_0\,(1+i\,r_0)^n}.$$

$$\frac{(1+i)^n - 1}{i + r\,i\,[(1+i)^n - 1]} = \frac{1}{i\,r_0} \cdot \frac{(1+i\,r_0)^n - 1}{(1+i\,r_0)^n}.$$

$$\frac{r\,[(1+i)^n - 1]}{(1+i)^n - 1} + \frac{1}{(1+i)^n - 1} = \frac{r_0\,(1+i\,r_0)^n}{(1+i\,r_0)^n - 1}.$$

Whence
$$r = \frac{r_0(1+i\,r_0)^n}{(1+i\,r_0)^n - 1} - \frac{1}{(1+i)^n - 1} \quad (13)$$

It will thus be seen that it is possible to get the same results using either Hoskold's formula or the ordinary one, provided that the risk factor is properly adjusted, this factor being naturally greater when the ordinary formula is employed, because a bigger annual income must be provided, so that a larger amount can be set aside at each period to form the sinking fund, this larger amount being required because the rate of interest at which it accumulates is by hypothesis lower according to Hos-

kold's formula than it is according to the ordinary one. In practice the above formula is far too cumbersome for use, and Hoskold's formula is very generally adopted. The only object of the above investigation is to show that there is no reason to suppose, as has often been done, that the ordinary single-rate formula is inapplicable to mining ventures, but that there is a risk ratio which, when applied to the ordinary formula, gives the same result as though Hoskold's formula with a different risk factor were employed. Unfortunately, it is impracticable to get a convenient expression for r_0 in terms of r, and this can only be calculated by a method of approximations; this constitutes an additional reason for working by Hoskold's formula, the application of which presents no difficulties, and which will accordingly be adhered to here.

As an example of the above, let it be required to calculate the actual value of an annuity of £100 per annum for 20 years, accumulating at 3 per cent., the risk factor to be 2·67, so that the capital is to earn 8 per cent. risk interest. Income tax is not taken into account. The actual value of such an annuity according to Hoskold's formula is £853·126; it will be found that this capital will purchase a twenty year annuity of £100·208 if a risk factor of 3·33 or a risk rate of 10 per cent. be allowed by the single rate method, so that, practically speaking, it would make no difference whether the calculation is made on the former or on the latter basis as long as the gilt-edged rate of interest is multiplied by the proper risk factor. According to the formula given above, the risk factor by Hoskold's method, corresponding to the risk factor of 3·33 per cent. by the one rate method, should be 2·6748, so that the two rates by Hoskold's method should be respectively 3 per cent. and 8·0244 per cent., to correspond with the result

obtained by a 10 per cent. single rate method. If, on the other hand, it be desired to determine what rate by the ordinary method corresponds to an accumulative rate of 3 per cent. and a risk rate of 8 per cent., the risk factor for the single rate method will be found by a method of approximations to be 3·3243, correct to three decimal places, so that the risk rate would be equal to 9·973 per cent.

Occasionally it is necessary to determine the actual value of an annuity increasing or decreasing in arithmetical or geometrical progression, when the calculation has to be made in accordance with the above principle—that is to say, when it involves both a risk rate and an accumulative rate of interest. As before, let the periodical increment be b, so that the annuity at the end of the

1st accounting period	=	a
2nd ,,	=	$a + b$
3rd ,,	=	$a + 2\,b$
nth ,,	=	$a + (n - 1)\,b$.

If A be the actual value, interest at risk rate = A s; the amount to be invested at the end of the 1st accounting period = $a - \mathrm{A}\,s$, which at the end of the term will amount at the ordinary rate to $(a - \mathrm{A}\,s)\,(1 + i)^{n-1}$; at the end of the 2nd period the sum to be invested = $a + b - \mathrm{A}\,s$, which amounts to $(a + b - \mathrm{A}\,s)\,(1 + i)^{n-2}$, etc.; at the end of the nth period the amount will be

$$a + (n - 1)\,b - \mathrm{A}\,s.$$

$$\mathrm{A} = (a - \mathrm{A}\,s)\,(1 + i)^{n-1} + (a + b - \mathrm{A}\,s)\,(1 + i)^{n-2} + (a + 2\,b - \mathrm{A}\,s)\,(1 + i)^{n-3} \ldots + a\,(n-1)\,b - \mathrm{A}\,s.$$

$$\mathrm{A}\,[1 + s + s\,(1 + i) \ldots + s\,(1 + i)^{n-1}] = a\,(1 + i)^{n-1} + (a + b)\,(1 + i)^{n-2} + (a + 2\,b)\,(1 + i)^{n-3} \ldots + a + (n - 1)\,b.$$

$$\text{Whence } A = \frac{a\,[(1+i)^n - 1] + \frac{b}{i}\,[(1+i)^n - 1] - n\,b}{i + s\,[(1+i)^n - 1]}$$

$$= \left(a + \frac{b}{i}\right) \frac{(1+i)^n - 1}{i - s + s\,(1+i)^n} - \frac{n\,b}{i - s + s\,(1+i)^n}$$

$$= \frac{a + \frac{b}{i}}{s + \frac{i}{(1+i)^n - 1}} - \frac{n\,b}{i - s + s\,(1+i)^n}. \quad . \quad . \quad (14)$$

The first part of this expression is the present value of an annuity of $a + \frac{b}{i}$ calculated by Hoskold's method at risk rate s and accumulative interest i per unit; $s\,(1+i)^n$ is the amount of s pounds at interest i for n years, so that the calculation of this expression may be facilitated by the use of tables.

Again, as before, let the annuity for the 1st accounting period be a, and let it increase geometrically by a common ratio k, so that the annuity for the 1st period is a, for the 2nd $a\,k$, for the 3rd $a\,k^2$, and for the nth $a\,k^{n-1}$.

At the end of the 1st period there is available for investment $a - A\,s$, producing at the end of term $(a - A\,s)(1+i)^{n-1}$.

At the end of the 2nd period there is available for investment $a\,k - A\,s$, producing at the end of term $(a\,k - A\,s)(1+i)^{n-2}$.

At the end of the 3rd period there is available for investment $a\,k^2 - A\,s$, producing at the end of term $(a\,k^2 - A\,s)(1+i)^{n-3}$.

At the end of the nth period there is available for investment $a\,k^{n-1} - A\,s$ producing at the end of term $a\,k^{n-1} - A\,s$.

A is equal to the sum of all these products, or

$$A = (a - A\,s)(1+i)^{n-1} + (a\,k - A\,s)(1+i)^{n-2} + (a\,k^2 - A\,s)(1+i)^{n-3} + \ldots\ a\,k^{n-1} - A\,s.$$

$$A + A\,s + A\,s\,(1+i) + \ldots A\,s\,(1+i)^{n-2} + A\,s\,(1+i)^{n-1}$$
$$= a\,(1+i)^{n-1} + a\,k\,(1+i)^{n-2} + a\,k^2\,(1+i)^{n-3} + \ldots a\,k^{n-1}.$$

$$A\left(1 + s\,.\,\frac{(1+i)^n - 1}{i}\right) = a\,(1+i)^{n-1}\,\frac{1 - \dfrac{k^n}{(1+i)^n}}{1 - \dfrac{k}{1+i}}.$$

$$A\left(\frac{1 + s\,[(1+i)^n - 1]}{i}\right) = a\,.\,\frac{(1+i)^n - k^n}{1 + i - k}.$$

Whence $A = a\,.\,\dfrac{(1+i)^n - k^n}{1+i-k}\,.\,\dfrac{i}{i + s\,[(1+i)^n - 1]}$. **(15)**

Decreasing annuities are dealt with in precisely the same manner as previously indicated.

Some of the problems on which the widest differences of opinion have arisen are those due to the deferment of the annuity for a certain length of time.

The present value P of an annuity a at rate of interest i per unit paid at the end of each of n accounting periods and deferred for d accounting periods, so that in fact the first payment is made at the end of $d + 1$ periods, may be obtained in several ways. A method frequently given is that of subtracting the actual value of a similar annuity payable for d periods from the actual value of one payable for $d + n$ periods.

$$\text{Or} \quad P = \frac{a}{i}\left[1 - (1+i)^{-\overline{d+n}}\right] - \frac{a}{i}\left[1 - (1+i)^{-\overline{d}}\right]$$
$$= \frac{a}{i}\,\frac{(1+i)^n - 1}{(1+i)^{\overline{d+n}}}. \quad . \quad . \quad . \quad . \quad . \quad \textbf{(16)}$$

Another method of solving the problem is the following:—Since A is the actual value of an an-

nuity a for n periods to commence at the end of d periods,

$$P = \frac{A}{(1+i)^d}$$

$$= \frac{1}{(1+i)^d}\left(\frac{a}{i}\frac{(1+i)^n - 1}{(1+i)^n}\right)$$

$$= \frac{a}{i}\frac{(1+i)^n - 1}{(1+i)^{n+d}}, \text{ as before.}$$

The latter method indicates that if for any reason a different rate of interest, such as i_0 per unit, is to be allowed during the period of deferment only, the above expression would become :

$$\frac{a}{i}\frac{(1+i)^n - 1}{(1+i)^n (1+i_0)^d}. \quad . \quad . \quad . \quad (17)$$

It also follows that, if interest during the period of deferment is payable x times per annum for d_1 years, the rate of interest being i_1 per unit per annum,

$$P = \frac{A}{\left(1+\frac{i_1}{x}\right)^{d_1 x}}. \quad . \quad . \quad . \quad (18)$$

In all cases where a single rate of interest is admissible, or where the problem can be reduced to a single rate during the annuity period, these expressions appear to be entirely free from objections.

There is, however, a considerable difference of opinion as to the correct method to be employed when two rates have to be taken into account during the term of the annuity.

The simplest case to be considered is where for any reason the mineral property will not come into value for a certain period of time ; for example, a colliery company, controlling a royalty that will take d years

to work out, wishes to acquire control of an adjoining royalty, which it cannot commence to work until the first one is exhausted, but which, nevertheless, could be worked independently by someone else, and to which, therefore, its full disposal value must be assigned. It is quite possible that it may be worth more to the colliery in question than to anyone else, but this consideration lies outside the province of the valuer, except in so far as he may have to take the relative costs of development into account. If A be the actual value of the royalty to be acquired, its present value is simply $\frac{A}{(1+i)^d}$ or if A is determined by Hoskold's formula based on an expected periodic revenue a, the accumulative and risk rates of interest being respectively i and s per unit, its present value will be :—

$$\frac{a}{(1+i)^d} \frac{1}{s+i[(1+i)^n-1]^{-1}}. \quad . \quad . \quad (19)$$

There is no justification for using the risk rate of interest allocated to mining for the term of deferment, because there is no mining risk. That risk has been properly, and, presumably adequately, provided for in determining the actual value. If the colliery company at the end of d years for any reason desire to sell this additional royalty instead of working it, the only price that could legitimately be demanded would be the actual value as above determined, and there is no reason why the company should receive a risk rate of interest on its original purchase. Or again, if the colliery company proposes to make the purchase at the end of d years, it would only need to set aside such a sum of money as would at the normal rate of interest amount to the actual value. The point must be emphasised that there is no justification for expecting a mining

risk rate of interest when no mining is being done. Possibly a rate a trifle higher than the normal rate might be expected on the grounds that it might not be possible to find a purchaser at the desired moment, and that the value of such property might go down in the interval, though it must be admitted that the latter argument cuts both ways. Nevertheless, it might be considered equitable to allow a rate somewhat above the normal gilt-edged rate of interest to cover the risk of the market.

The case is different when the term of deferment to be considered is the length of time that must elapse between the commencement of mining operations and the stage of remunerative production, because in this case the full mining risk is operative during the period of deferment. Various solutions of this problem have been put forward, and two of them at any rate are adopted by mining engineers, and may be briefly discussed.

Hoskold has dealt with the problem as follows (substituting for the sake of uniformity the notation here employed for that which he himself uses) :—*

" If instead of an annuity of a pounds being entered upon immediately, it can only be realised at the end of the dth year, and to continue n years thereafter, the purchaser will realise s per cent. on his outlay P during the whole term of $d + n$ years ; and here, as was done in the last preceding case, he can invest the surplus annuity only at the rate of i per cent."

(Mr. Hoskold has written " per cent.," but his calculations show that he really means per unit as in previous formulas.—H.L.)

" It is necessary, therefore, to determine the relation

* " Engineer's Valuing Assistant," by A. D. Hoskold. Second Edition, p. 12.

existing between P and a, and, as it may be seen that no annuity can be paid during the deferred term of d years, P would accumulate or amount at the end of the dth year to $P(1 + s)^d$. When, however, the annuity is entered upon, which as a matter of course it would be at the end of the dth year, it is, as previously shown, separable into two parts, that is to say, first a year's interest on the amount which the purchase money P has now attained, namely, $P\,s\,(1 + s)^d$; and secondly, $a - P\,s\,(1 + s)^d$, the sum which must be invested at the rate i per cent. and which will reproduce $P(1 + s)^d$ at the end of $d + n$ years. Then we have

$$[a - P\,s\,(1 + s)^d]\,M_n = P(1 + s)^d.$$

Solution of this equation gives

$$P = a\,\frac{M_n}{(1 + s)^n\,(1 + s\,M_n)}, \text{ and}$$

$$a = \frac{P(1 + s)^d\,(1 + s\,M_n)}{M_n},$$

in both of which M_n denotes the amount of an annuity of £1 in n years."

Substituting for M_n its value $\frac{(1 + i)^n - 1}{i}$, the expression for P becomes

$$\frac{a}{(1 + s)^d}\,\frac{(1 + i)^n - 1}{i + s\,[(1 + i)^n - 1]}.$$

It will be seen that this expression for the present value is the actual value as determined by Hoskold's two-rate annuity formula, allowing the present value to accumulate at the risk rate s. This, however, implies that not only does the present value earn interest at the risk rate s, but that the interest upon this capital also accumulates at the same risk rate; as already shown, the only justification for expecting to receive a risk rate upon any sum is that it is itself invested in a

speculative undertaking. It might conceivably be argued that, since no interest is actually paid during the deferment period, and that since the return of both the capital and the accumulating interest depends upon the success of the mine, the interest is in effect invested in a speculative undertaking, but it is now generally held that this fact is no justification for assuming interest on capital invested at a risk rate to itself accumulate at the same rate, and indeed Hoskold's annuity formula depends entirely for its validity upon the correctness of this view. The writer accordingly looks upon this method of deferment as inconsistent with the use of Hoskold's annuity formula, and has rejected it in favour of the following line of reasoning :—

A purchaser invests a sum of money P upon the understanding that there will be an unproductive term of deferment of d accounting periods, that at the expiration of this term there will be a productive term of n periods, during which his investment will receive revenue at the rate of a per period, that his investment during the entire term of $d + n$ periods shall be deemed to be earning interest at the risk rate of s per unit, that interest during the term of deferment, d, shall be deemed to accumulate at the rate of i per unit, and that he shall be able to form a sinking fund for the redemption of this additional capital plus any accumulated interest thereon not paid at due date at the same accumulative rate of i per unit. In other words, the sum P is to be the present value of a series of n annuities, a, deferred for d accounting periods upon the condition that the investment shall throughout—*i.e.*, for $d + n$ periods—earn the risk rate of interest s, whilst all interest is deemed to accumulate at the normal accumulative rate i.

Hence the revenue or annuity a for n accounting

periods must equal the amount of P under the above conditions at the end of $d + n$ accounting periods.

The amount of n annuities of a at rate i

$$= \frac{a}{i}[(1 + i)^n - 1];$$

at the end of $d + n$ periods the investor should be in possession of his original outlay P plus the interest P s accumulated at the rate i for $d + n$ periods, or

$$P + \frac{P\,s}{i}[(1 + i)^{d+n} - 1] = \frac{P}{i}[i + s((1 + i)^{d+n} - 1)].$$

Equating $\frac{a}{i}[(1 + i)^n - 1] = \frac{P}{i}[i + s((1 + i)^{d+n} - 1)].$

Whence
$$P = a\frac{(1 + i)^n - 1}{i + s[(1 + i)^{d+n} - 1]}. \quad . \quad (20)$$

This result is the same as that ultimately put forward by Mr. T. A. O'Donahue,* and is also equivalent to that arrived at by Mr. George King,† and quoted and accepted by Sir R. A. S. Redmayne and Mr. Gilbert Stone in their work ‡ on the subject; the above method is a direct one for arriving at it, and shows clearly the mode of its derivation and the respect in which it differs from Hoskold's expression.

Mr. O'Donahue § derives the expression given above —namely, No. 20—from the consideration that the amount of the annuity at the end of the term should be equal to the actual value together with its accumulated interest for $n + d$ years, or

$$P\left\{1 + \frac{s}{i}[(1 + i)^{n+d} - 1]\right\} = a\frac{(1 + i)^n - 1}{i}.$$

Whence $P = a\frac{(1 + i)^n - 1}{i + s[(1 + i)^{n+d} - 1]}$, precisely as obtained above.

* "The Valuation of Mineral Property," p. 43.

† "The Theory of Finance," p. 38.

‡ "The Ownership and Valuation of Mineral Property," p. 130.

§ "The Valuation of Mineral Property," p. 43.

Here again it may become necessary to consider the effect of two different fractional periods of the year, accumulative interest being payable at one set and the annuities payable at another set of intervals. Thus, if it be assumed that the annuity of a_1 per annum, and, with it, the risk rate of interest s_1 are payable y times in the year for a term of n_1 years to commence at the end of a term of deferment of d_1 years, whilst interest at the safe rate i_1 is payable x times in the year, the annuity amounts at the end of the term as before to

$$\frac{a_1}{y} \cdot \frac{\left(1+\frac{i_1}{x}\right)^{n_1 x}-1}{\left(1+\frac{i_1}{x}\right)^{\frac{x}{y}}-1} \text{ (see p. 163),}$$

whilst the principal at the end of $n_1 + d_1$ years will amount to

$$\mathrm{P}\left(1+\frac{s_1}{y}\,\frac{\left(1+\frac{i_1}{x}\right)^{(n_1+d_1)x}-1}{\left(1+\frac{i_1}{x}\right)^{\frac{x}{y}}-1}\right)$$

Equating

$$\mathrm{P}\left(\frac{y\left[\left(1+\frac{i_1}{x}\right)^{\frac{x}{y}}-1\right]+s_1\left[\left(1+\frac{i_1}{x}\right)^{(n_1+d_1)x}-1\right]}{y\left[\left(1+\frac{i_1}{x}\right)^{\frac{x}{y}}-1\right]}\right)$$

$$=\frac{a_1\left[\left(1+\frac{i_1}{x}\right)^{n_1 x}-1\right]}{y\left[\left(1+\frac{i_1}{x}\right)^{\frac{x}{y}}-1\right]},$$

$$\text{or } \mathrm{P}=\frac{a_1\left[\left(1+\frac{i_1}{x}\right)^{n_1 x}-1\right]}{y\left[\left(1+\frac{i_1}{x}\right)^{\frac{x}{y}}-1\right]+s_1\left[\left(1+\frac{i_1}{x}\right)^{(n_1+d_1)x}-1\right]}. \quad (21)$$

If the number of times per annum is the same in both cases, or $x = y$, this expression becomes :—

$$P = \frac{a_1\left[\left(1+\frac{i_1}{x}\right)^{n_1 x}-1\right]}{i_1+s_1\left[\left(1+\frac{i_1}{x}\right)^{(n_1+d_1)x}-1\right]}. \quad . \quad . \quad (22)$$

Here again the symbols a_1, i_1, s_1, d_1 and n_1 refer to years, and not to periods, and the formula is equivalent as before to No. 20.

The above formulas are applicable to the case in which an annuity commences as from a definite date and then continues uniformly ; they can, therefore, legitimately be applied to income derived from ownership or control of minerals, which can be leased so as to bring in a royalty, the amount of which can be estimated, whilst the date from which the income commences to run is either definitely known or can be assumed on reasonable grounds. They are, however, not applicable without some qualification to a mining venture, in which the annuity is taken to be the probable average profit derivable from the mining operations. This probable profit can be estimated, but, as already indicated, a mine does not immediately produce the full average profit as soon as it enters upon the period of production. In a mining venture there are three definite stages that can be distinguished.

I. The preparatory stage, during which shafts are sunk or adits driven, plant, houses, etc., constructed, and a certain amount of development work done. In this stage there is no profit possible ; a certain amount of mineral may be won in the course of development work, but any money realised by its sale can only be taken as to this extent diminishing the outlay necessarily incurred. It is part of the mining engineer's duty

to draw up estimates of the amount of such capital outlay that will be required in any given case, and which should include the necessary working capital. Since the valuation of the mine as a going concern is based on the actual values of the revenue expected to be derived from working it for a term equal to the life of the mine or to the length of the lease as the case may be, this value includes the whole of the capital expenditure during the preparatory stage. To get the present value of the mining property at the commencement of this stage, the capital outlay must be deducted from the present value at the commencement of mining operations determined as above. If this outlay is to be spread over a number of years, the present value of the presumed annual outlay must be determined as at the commencement of the preparatory operations, and it is this present value that must be deducted from the calculated gross present value of the going mine in order to get the net present value of the mining property.

II. The next stage is one that commences from the time that mineral can be won at a profit, and continues until the full expected annual production is attained. This stage will in most cases include also intensive development, which is dealt with as previously indicated (see p. 109); the net result is that during this stage the mine makes continuously increasing profits as it shows progressively greater outputs until the full output is reached. These increasing profits may be dealt with in various ways. If the anticipated profits can be assumed to increase in arithmetical or geometrical progression, the actual value of these progressive profits can be calculated by the help of the formulas already investigated; if the profits must be assumed to increase irregularly, which is the more general case, the actual value as at the commencement of the present stage

must be calculated separately for each year or for each accounting period. As this stage is generally a relatively short one, a less accurate method is frequently adopted, the stage of progressive profits being disregarded, the preparatory stage being assumed to last somewhat longer than it really should, and the productive stage to commence somewhat earlier. Thus, if it is calculated that a mine will require three years for the preparatory work, that at the end of these three years it will commence to make a small profit, and that this will gradually increase until at the end of four years more it will have reached its full expected output and will be making its full estimated profit, many engineers would calculate the preparatory period as five years, and take the full output as though commencing after the end of the fifth year. As a general rule, this method may be adopted without serious error; it must not be forgotten that the valuer is working only on probable estimates, which he knows to be affected by a more or less considerable probable error, and when the data for the calculations are necessarily approximations, excessive refinement in the mathematical handling of these data is quite out of place.

III. The third stage is that in which the mine has reached its full expected output, and can, therefore, be expected to be making the probable profit which forms the basis of the annuity calculation, the latter being worked out by means of the formulas already discussed.

In practice it is generally found more convenient to work backwards, commencing at the third stage and using the annuity expected during its continuance as the basis of the calculation. This gives the actual value as at the commencement of the full productive stage, and its present value must be calculated as at

the period of commencing mining operations. To this must be added the present value at the same moment of the profits expected to be obtained in the stage of progressive production, and from the sum thus obtained must be subtracted the present value of the capital outlay required to bring the property into the profit-earning stage. The difference is obviously the present value of the mineral property looked upon as a working mine.

A difficult question, which different engineers solve in various ways, is that of the residual value of the plant and equipment. It quite often happens that, when the working life of a mine is finished, the plant and equipment are practically worthless, especially if the mine is situated in a district difficult of access ; in such a case it may well occur that the cost of lifting the plant and equipment and of getting it to a market may be more than the material would fetch, in which case the value is, of course, nil ; in other cases, especially in the case of a short-lived mine situated in the centre of a mining district, the sale of plant and equipment might realise a sum of sufficient importance to be worth taking into account. This sum may be looked upon as a reimbursement of a portion of the capital outlay ; it does not diminish the required outlay, because the latter has to be expended before the mine can be worked ; furthermore, it is only realised after the annuities derived from the profits of the mining operations have ceased ; it cannot, therefore, be taken as increasing in any way the value of the mineral property. It may perhaps best be taken into account in this way :—

It has been seen that the annuity may be considered in part as interest upon the capital invested, and in part as a sinking fund, which is to reproduce this capital at the end of the term ; consider that a portion of the

capital to be thus amortised is in effect repaid by the sum realised by the sale of the plant.

Thus assume that at the end of the term the plant and equipment fetch a sum of money which may be conveniently expressed as equal to $\frac{P}{c}$ pounds; hence the n annuities need only amount to $P\frac{(c-1)}{c}$ instead of P, and the formula (No. 20) becomes :—

$$P = \frac{a\left[(1+i)^{n-1}-1\right]}{\left(\frac{c-1}{c}\right)\left(i+s\left[(1+i)^{d+n}-1\right]\right)}. \quad . \quad (23)$$

Strictly speaking, there is a fourth stage in the life of a mine—namely, when the output and the profit realisable therefrom commence to decline until ultimately the mine is no longer workable at a profit, when it is looked upon as exhausted. If desired, the present value of the stage of decreasing income could also be taken into the calculation upon the principles above enunciated, but in fact this is only exceptionally done. It is very rarely, if ever, possible to predict the life of a mine or the length of this fourth stage with any approach to accuracy; in any case the present value of the diminishing income obtainable during this stage after a tolerably long life is necessarily very small, and may safely be disregarded, the life of the mine being assumed to terminate when the quantity of mineral calculated to exist therein is supposed to be worked out at the normal rate of working. There are exceptional cases when the boundaries of a mineral deposit are accurately known, and when it can be foreseen that the life of the mine must be a very short one, and that the stage of diminishing output is relatively important as compared with the period of full production. Should

such cases arise, the calculations present no difficulty using the formulas that have been given previously.

It frequently happens that at the end of the working period, whether brought about by the exhaustion of the mine or the termination of the lease, the party working the mine is under certain liabilities, such as that of restoring the surface, filling up shafts, removing waste heaps, closing adit mouths, etc ; the cost of these can but rarely be estimated beforehand, but in any case provision must be made for them. The usual plan is to estimate or guess them at a certain sum, and this is then taken as being a deduction from the sum realised by the sale of the plant ; if they exceed the latter, the difference must be added to the amount that has to be provided for by means of the sinking fund. When neither the final value of the plant nor the probable cost of the liabilities at closing down is known, it is quite usual to consider that the one will about off-set the other, and in most cases no serious error is involved in this assumption.

A problem which sometimes arises indirectly in the course of mineral valuations is connected with the redemption of loans. Thus, a mining company may borrow money on debentures or otherwise, and undertake to set aside annually a definite sum in order to redeem a certain portion of the loan each year and to pay interest at an agreed rate upon the outstanding portion. The relation between the capital borrowed and the annual payment required to cover both interest and redemption is the simple annuity formula (No. 4), $A = \frac{a}{i} \cdot \frac{(1+i)^n - 1}{(1+i)^n}$, A in this case representing the capital borrowed ; the application of the formula is indicated in the third method given on p. 155 for arriving at the formula in question. The first step is the calcu-

lation of the annuity corresponding to a given loan. Then, as indicated on p. 155, a sum equal to $a - \mathrm{A}\,i$ can be devoted to the redemption of the loan at the end of the 1st accounting period; at the end of the 2nd accounting period the reduction is equal to

$$(a - \mathrm{A}\,i)\,(1 + i),$$

at the end of the 3rd accounting period it is

$$(a - \mathrm{A}\,i)\,(1 + i)^2,$$

and so on; since $(1 + i)$, $(1 + i)^2$, etc., are the amounts of one pound at interest i per unit at the end of 1, 2, etc., accounting periods, these can conveniently be taken out from any set of Actuarial Tables; the calculation is thus somewhat shortened, though in practice it is generally just as easy to work it out logarithmically.

It is often preferable to adopt another method, which, though very simple, is somewhat more laborious when it has to be applied to a long series of accounting periods. This consists in calculating for each accounting period the interest payable therein; this is subtracted from the annuity, and the difference subtracted from the amount of the outstanding loan gives the amount outstanding at the end of the period in question. In this way a table is drawn up showing for each period the interest payable, the amount applied to the reduction of the borrowed capital and the capital outstanding. As an example, assume that a sum of £10,000 is borrowed, which is to be repaid by a series of payments during the course of ten years, the outstanding portion of the loan to bear interest at 6 per cent. per annum.

Then
$$a = \frac{10{,}000 \times 1{\cdot}06^{10} \times 0{\cdot}06}{1{\cdot}06^{10} - 1}$$

$$= £1{,}358{\cdot}68.$$

As the interest payable at the end of the first year is £600, £1,358·68 – 600 or £758·68 is available for redemption, so that the outstanding liability is reduced to £9,241·32. Interest on this latter sum for the next year amounts to £554·48, leaving £804·20 available for redemption, and again reducing the outstanding liability to £8,437·12.

The calculation is continued in this way for each year, and a redemption table is drawn up as follows :—

End of Year.	I. Interest Payable.	II. For Reduction of Liability.	III. Liability Outstanding.
1	£600·00	£758·68	£9,241·32
2	554·48	804·20	8,437·12
3	506·23	852·45	7,584·67
4	455·06	903·62	6,681·05
5	400·86	957·82	5,723·23
6	343·39	1,015·29	4,707·94
7	282·48	1,076·20	3,631·74
8	217·90	1,140·78	2,490·96
9	149·46	1,209·22	1,281·74
10	76·90	1,281·78	0

The sum of each horizontal line in Columns I. and II. is necessarily always equal to the annuity, and the last figure but one in Column III. ought to be equal to the last figure in Column II. ; the small (and unimportant) discrepancy in the above calculation is due to its having been carried to only two decimal places ; when greater accuracy is required seven-figure logarithms should be used and the calculation carried to three or four places of decimals, but in any case, the sum to be redeemed each year is always rounded off to the nearest convenient unit.

The various formulas discussed in this chapter

should cover all the ordinary cases that occur in mineral valuation. Here and there special cases will arise owing to unusual conditions being required; it is impossible to provide for all such cases by any series of set formulas, but care has been taken to show in every instance how the formula has been derived; in this way the reader should have no difficulty in modifying any formulas to suit any specific case that may arise and which is not covered by the general expressions here given.

In order to indicate the application of these formulas, a few examples will be given in the next chapter, and will be worked out in detail with the object of showing exactly how these formulas can be most conveniently applied in practice.

CHAPTER VI.

EXAMPLES.

In the present chapter a series of examples will be given showing the application of the formulas investigated in Chapter V. These will be fully worked out so as to show both the method of applying the various formulas and the process of calculation which the author recommends. For this purpose the calculations are set out in detail. It should be premised that the examples given are wholly imaginary, although care has been taken to devise conditions that might readily occur in practice. At the same time, attention has been directed mainly to the preparation of examples which are intended to bring out the application of the formulas in question rather than to reproduce exactly the conditions met with in practice. Advantage has also been taken of these examples in order to introduce discussions on various points that are likely to occur in the course of valuations, and which cannot be settled by reference to a formula.

I. It is proposed to purchase the lease of a coal royalty, 1,250 acres in extent; it contains one seam of coal, and borings and trial pits show that its probable average thickness is 5 feet 7 inches $\pm$ 5 inches. The coal is a steam coal of reasonably good quality, and its specific gravity has been found to be 1·3. It is estimated that an allowance of 20 per cent. of the calculated tonnage of coal must be made for barriers, coal to be left unwrought, loss in working, and possibly for as

yet unknown "wants." The existing tonnage is accordingly calculated as $1{,}250 \times 1{,}215 \times 1{\cdot}3 \times 5{\cdot}583 \pm 1{,}250 \times 1{,}215 \times 1{\cdot}3 \times 0{\cdot}4167 = 11{,}023{,}590 \pm 822{,}655$ tons, from which the above-mentioned 20 per cent. must be deducted, leaving a tonnage of workable coal calculated at $8{,}818{,}870 \pm 658{,}125$, or between 8,160,745 and 9,476,995 tons. The probable error in tonnage is equal to 7·5 per cent.

For certain reasons it is not proposed to commence operations until after the end of five years. It is estimated that preparatory operations will take three years, and it is calculated that these operations, the necessary plant, and the work of development, together with an allowance for working capital, will require an outlay of £400,000, of which £100,000 will be required in the first year, £140,000 in the second year, and £160,000 in the third year. It is estimated that after the end of the third year remunerative mining will commence, and that the annual outputs and gross profits per ton will be as follows :—

1st year,	100,000 tons,	. . .	6d. per ton.
2nd ,,	200,000 ,,	. . .	1s. ,,
3rd ,,	300,000 ,,	. . .	2s. ,,
4th ,,	400,000 ,,	. . .	3s. ,,

From the fifth year onwards it is proposed to work at the rate of 500,000 tons per annum, and the estimated gross profit is 4s. per ton. Out of the above output, a certain proportion will be required for colliery consumption, only the balance being available for sale ; in this example, however, the profits are calculated on the tonnage of the total raisings and not on the vend, so that the question of colliery consumption can here be disregarded, because the annual income can be calculated without reference to it.

It is required to determine what is the present value of the mining property, or, in other words, what sum a purchaser can afford to give for the lease, taking the normal gilt-edged rate of interest at 4 per cent. per annum, and allowing a risk factor of $2\frac{1}{2}$—that is to say, expecting to get a risk rate of interest upon the whole investment at the rate of 10 per cent. per annum, assuming the interest in both cases to be paid half-yearly. The valuer takes the view—perhaps the somewhat pessimistic view—that it is not safe to calculate upon income tax falling below its present figure during the life of the mine, so that, allowing for income tax at 5s. in the £, the accumulative rate will be $1\frac{1}{2}$ per cent. per half-year, and the risk rate 3·75 per cent. per half-year—that is, $i = 0{\cdot}015$ and $s = 0{\cdot}0375$.

The valuer considers himself justified in calculating upon 8,500,000 tons of coal, and deducting 1,000,000 tons to be wrought during the stage of progressive output, the life of the colliery at full output will be taken as 15 years. During these 15 years the annual gross profit will be 500,000 tons at 4s., or £100,000; allowing for income tax at 5s. in the £, the profit will be £75,000 per annum, or £37,500 per half-year. There will be 30 half-yearly periods of full production, and the commencement of the full productive period will be deferred 3 + 4 or 7 years or 14 half-yearly periods from the commencement of mining operations, during the whole of which time the mining risk is, therefore, being incurred.

It would be possible to employ either Formula 22 or Formula 20; in the former case the calculation would be made taking the value of i_1 as 0·03, s_1 as 0·075, n_1 and d_1 as 15 and 7 respectively, and a_1 as £75,000. The figures given above are adapted to use with Formula 20, namely :—

$$A = \frac{a\,[(1+i)^n - 1]}{i + s\,[(1+i)^{n+d} - 1]} \text{ or}$$

$$A = \frac{37{,}500\,(1{\cdot}015^{30} - 1)}{0{\cdot}015 + 0{\cdot}0375\,(1{\cdot}015^{44} - 1)}.$$

The calculation is as follows, using five-figure logarithms :—

Log 1·015	=	0·00647		
		44		
	Log^{-1}	0·28468	=	1·9261
				−1
				0·9261
Log 0·9261	=	$\bar{1}$·96666		
Log 0·0375	=	$\bar{2}$·57403		
	Log^{-1}	$\bar{2}$·54069	=	0·034728
				+0·015
				0·049728
Log 1·015	=	0·00647		
		30		
	Log^{-1}	0·19410	=	1·5635
				−1
				0·5635
Log 37,500	=	4·57403		
Log 0·5635	=	$\bar{1}$·75089		
		4·32492		
−Log 0·049728	=	− $\bar{2}$·69660		
	Log^{-1}	5·62832	=	£424,930.

If the calculation be made with seven-figure logarithms, the value obtained for A will be found to be £424,855. It will be generally found that five-figure logarithms give a quite sufficiently close approximation, except, however, that when the logarithm of such an expression as $1 + i$ has to be multiplied by a large exponent, it is an advantage to use 6- or 7-figure logarithms, in order to get the first five figures of the product correct. Using this method, which the writer prefers and will use in subsequent calculations, the result would be £424,865. Alternatively the result may be taken from ordinary interest tables. For example, in the above calculation $1{\cdot}015^{44}$ worked out by 7-figure logarithms is 1·92533, or it might be taken from the ordinary interest tables, where the amount of one pound in 44 years at $1\frac{1}{2}$ per cent. will be found to be 1·92533 pounds.

To the above value must be added the present value of the income obtained during the four years of progressive production. This income works out as follows :—

1st year,	£2,500	gross =	£1,875,	less Income Tax.
2nd ,,	10,000	,, =	7,500,	,, ,,
3rd ,,	30,000	,, =	22,500,	,, ,,
4th ,,	60,000	,, =	45,000,	,, ,,

The value of these four sums at the commencement of mining operations must be determined. As they form neither a geometrical nor an arithmetical series, the present value of each must be determined separately and the four added together. The conditions of the problem do not give any information as to what proportion of the annual income is expected to be obtained in each half-year, so that the valuer has several alternative methods of calculation ; he may either assume

what the income in each half-year will be, taking what he may consider to be a reasonably probable rate of progressive increase, he may adopt Formula 21, or he may use the figures as given for each year, and substitute equivalent rates of interest payable yearly instead of the rates of $1\frac{1}{2}$ per cent. and $3\frac{1}{4}$ per cent. per half-year, for which the original conditions stipulated. None of these methods is quite strictly accurate, but the latter is probably preferable, though the result will be a little on the low side, seeing that the profits will be realised throughout each working year, and not all at the end of the year.

Using Formula No. 1, $x \log (1 + j) = \log \left(1 + \frac{i}{x}\right)$; in this case $x = 2$, hence rates of interest to be used are respectively $\log^{-1} (2 \log 1{\cdot}015) - 1$ and $\log^{-1} (2 \log 1{\cdot}0375) - 1$, or 3·023 per cent. for the accumulative rate and 7·64 per cent. for the risk rate. The first amount, £1,875, is due at the end of four years from the date of commencing operations, the second at the end of five years, and so on.

Using Formula 3, the expression to be calculated for the amount expected to be realised in each of the four years is :—

1st year, $\dfrac{1{,}875 \times 0{\cdot}03023}{0{\cdot}03023 + 0{\cdot}0764\,(1{\cdot}03023^{4} - 1)}$

2nd year, $\dfrac{7{,}500 \times 0{\cdot}03023}{0{\cdot}03023 + 0{\cdot}0764\,(1{\cdot}03023^{5} - 1)}$

3rd year, $\dfrac{22{,}500 \times 0{\cdot}03023}{0{\cdot}03023 + 0{\cdot}0764\,(1{\cdot}03023^{6} - 1)}$

4th year, $\dfrac{45{,}000 \times 0{\cdot}03023}{0{\cdot}03023 + 0{\cdot}0764\,(1{\cdot}03023^{7} - 1)}$

The calculation for the first year, again using five-figure logarithms, is as follows :—

Log	1·03023 =	0·12934		
		4		
		Log^{-1} 0·05174 =		1·1265
				−1
				0·1265
Log	0·1265 =	$\bar{1}$·10175		
	0·0764	−$\bar{2}$·88309		
		Log^{-1} $\bar{3}$·98484 =		0·009657
				+0·03023
				0·039887
Log	1875 =	3·27300		
Log	0·03023 =	$\bar{2}$·48044		
		1·75344		
−Log	0·039887 =	−$\bar{2}$·60083		
		Log^{-1} 3·15261 =		£1,421.

The calculations for the other three years are made in the same way, and the results for the four years are as follows :—

1st year, ...	£1,421
2nd ,, ...	5,335
3rd ,, ...	15,057
4th ,, ...	28,377
Total . .	£50,190

This total added to the figure previously obtained gives a sum of £475,120 as the value of the proposition

at the time of commencing operations, including the value of the capital that has to be spent during the first three years. Provision has been made in the calculation for payment of interest upon this capital at the risk rate from the date that this capital commences to be employed in mining operations, hence it is now a question of how much money must be deposited in the Bank to provide for disbursements.

Let it be assumed that the money is to be spent at an approximately uniform rate throughout the year, so that it will be approximately correct if the calculations were made as though the whole of each disbursement were made in the middle of the year in question; the capital to be provided at the commencement of mining operations could, therefore, be met by providing a sum of money equal to the present value of:—

£100,000 due in $\frac{1}{2}$ year at $1\frac{1}{2}$ per cent. per half-year, plus
£140,000 ,, $1\frac{1}{2}$ years ,, ,, ,,
£160,000 ,, $2\frac{1}{2}$,, ,, ,, ,,

This is equivalent to $\frac{100,000}{1{\cdot}015} + \frac{140,000}{1{\cdot}015^3} + \frac{160,000}{1{\cdot}015^5}$.
These can either be calculated out or they can be taken from Interest Tables showing the present value of one pound at $1\frac{1}{2}$ per cent. for one, three, and five years respectively. The results are

£98,522 + £133,885 + £148,522 = £380,929.

The value of the mining proposition at the date of commencing operations would, therefore, be £475,120 − £380,929, or £94,191. Under the specific conditions of the case, mining operations are to be postponed for five years, so that the above sum could be provided

by depositing in the bank a sum that will amount to the above figure in five years, allowing as before interest at the rate of $1\frac{1}{2}$ per cent. per half-year, or $\frac{94,191}{10 \cdot 15^{10}}$. The result may be obtained by logarithmic calculation or from Interest Tables, which show that £1 will amount in ten years to 1·16054 at $1\frac{1}{2}$ per cent.; hence $\frac{94,191}{1 \cdot 16054}$ = £81,161. This is, therefore, the sum of money that could equitably be paid for the lease in question under the conditions originally stated.

It was shown that the probable error in the quantity of available coal in the property was 7·5 per cent. This is a possible error which may affect the value of the mine as at the commencement of operations, and, therefore, the capital sum found above of £475,120. The value of this sum at the commencement of the period of postponement is $\frac{475,120}{1 \cdot 015^{10}}$ or £409,400, 7·5 per cent. of which is £30,705; the value might, therefore, fairly be stated as £81,196 ± £30,705. The probable error is relatively high, because the capital outlay on plant, etc., bears an abnormally large proportion to the value of the mineral, due largely to the short life assumed for the colliery. It need hardly be said that in practice a considerably cheaper equipment would be employed for so relatively low a total tonnage, but, as already explained, the conditions were selected to show methods of calculation rather than such as might normally be expected to prevail.

If the lease carries a dead-rent the present value of the money that would have to be disbursed by way of such dead-rent must be deducted from the value as above obtained. It is somewhat of an open question what rate of interest ought to be employed in arriving

at this present value. Perhaps the safest plan is to consider how much money would have to be deposited in a bank forthwith in order to pay the dead rent as it comes due, until the mine reaches a productive stage, and, therefore, to use an accumulative rate of interest as the basis of the calculation. If, for example in the present case, a dead rent were levied amounting to £500 per annum, payable half-yearly as from the date of purchase—that is to say, including the five years in which no mining work at all is being done, it would, therefore, have to be paid during eight years until a productive stage is reached ; the sum required would, therefore, be the present value of sixteen payments of £250, each at $1\frac{1}{2}$ per cent. interest per half-year, since this money will have to be paid during the five years in which the property lies idle and during the three years occupied in preparatory operations. The present value of these payments in accordance with Formula 4 is $\frac{250}{0{\cdot}015}\left(1 - \frac{1}{1{\cdot}015^{10}}\right) = £3{,}533$, and this accordingly is the amount to be deducted from the price as above determined.

II. If, on the other hand, the proposition to be valued were the purchase outright of the mineral property in question, the problem must be attacked from an entirely different side. The vendor is naturally enough not concerned as to what the purchaser will do with the property after he has bought it ; to the former it represents the present value of the unsevered coal as it lies in the ground, and the valuation must be conducted purely on that basis, so that the first point that arises is the determination of the value per ton of the unsevered coal at the moment of winning it, or, in other words, what is a fair royalty to allow for it. This must be determined by a number of conditions, such as the

quality of the coal, the thickness of the seam, its workability in the sense of lying at an easy gradient and being free from faults and other disturbances, the demand in the district for coal of this quality, its accessibility, the cost of development, its nearness to the markets, etc. ; it will generally be found that the royalty value of each seam in a district is fairly well determined by general consent ; it will fluctuate within certain well-defined limits, and its final value will generally be arrived at as the result of a bargain between the two parties. Let it be assumed that in the present case 6d. per ton is taken to be a reasonable royalty, this figure being based upon the royalty currently paid in the district upon coal of similar quality, occurring under approximately similar conditions and situated at approximately the same distance from its market. In the previous problem this royalty, although not specified, had in fact entered into the calculation as an item in the cost of getting the coal, and the profit reckoned upon was the selling price less the sum of the working costs together with the royalty ; if the company working the coal had also owned the royalty instead of leasing it, their profits would be higher by 6d. per ton, but this increase of profit would have to be applied to pay interest and redemption on the purchase price of the royalty.

In the present case the valuer would probably neglect the stage of progressive output ; he would value on the basis of a total quantity of 8,500,000 tons of workable coal and an output of 500,000 tons per annum, or a working life of seventeen years, allowing a deferment of five years for preparation and development. As a general rule, in a colliery lease there is an allowance of a reasonable quantity of coal burnt for the purposes of the colliery itself, no royalty being charged on the

quantity allocated to colliery consumption. In the same way, in purchasing coal in the ground, it is customary not to pay for the coal consumed in this way. A reasonable allowance for colliery consumption might be 6 per cent., so that the annual output subject to royalty would be 470,000 tons, and the purchase price ought to be the present value of the royalties upon this tonnage. A reasonable risk factor would be 1·5, so that if the gilt-edged rate be as before 4 per cent., the risk rate of interest would be 6 per cent. ; the valuer considers that income tax for the period in question is likely to range between 5s. and 3s. in the £, and averages it at 4s. The net accumulative rate of interest is, therefore, taken as $\frac{20-4}{20} \times 4$ or 3·2 per cent. The risk rate, allowing for mineral rights duty at 1s. in the £ is $\frac{16}{20} \times \frac{19}{20} \times 6$, or 4·56 per cent.—that is to say, that a man getting a net return of 4·56 per cent. on income derived from mineral rights after paying income tax and mineral rights duty is, in fact, getting gross interest at the rate of 6 per cent.

Assuming that it has been agreed between the parties that royalty shall be payable half-yearly, the half-yearly income from royalties is $\frac{16}{20} \times \frac{19}{20} \times 235{,}000$ at 6d., or £4,465. The purchaser has further undertaken that he will extract coal at a rate capable of paying the full royalty before the end of five years, so that a five years' deferment must be allowed for.

The present value in accordance with Formula 20 is, therefore :—

$$\frac{4{,}465\,(1{\cdot}016^{34}-1)}{0{\cdot}016 - 0{\cdot}0228\,(1{\cdot}016^{44}-1)}.$$

The calculation, using five-figure logarithms (except for log 1·016), is as follows :—

Log 1·016	= 0·006894		
	44		
Log^{-1}	0·30334	=	2·0106
			−1
			1·0106
Log 1·0106	= 0·00458		
Log 0·0228	= $\bar{2}$·35973		
Log^{-1}	$\bar{2}$·36431	=	0·02314
			+0·016
			0·03914
Log 1·016	= 0·006894		
	34		
Log^{-1}	0·23440	=	1·7155
			−1
			0·7155
Log 0·7155	= $\bar{1}$·85461		
Log 4465	= 3·64982		
	3·50443		
−Log 0·03914	= $\bar{2}$·59262		
Log^{-1}	4·91181	=	£81,622.

To this must be added the actual value of the five years' dead rent, or ten half-yearly payments of £250, which, less income tax and mineral rights duty as above, will leave net £190.

Allowing in this case the same rate of interest for

dead rents as was taken for the royalties, the actual value by Formula 10 will be:—

$$\frac{190\,(1{\cdot}016^{10} - 1)}{0{\cdot}016 + 0{\cdot}0228\,(1{\cdot}016^{10} - 1)},$$

and the calculation is as follows:—

Log 1·016	=	0·006894	
		10	
	Log^{-1}	0·06894	= 1·1720
			—1
			0·1720
Log 0·1720	=	$\bar{1}$·23553	
Log 0·0228	=	$\bar{2}$·35973	
	Log^{-1}	$\bar{3}$·59526	= 0·003938
			+0·016
			0·019938
Log 1720	=	$\bar{1}$·23553	
Log 190	=	2·27875	
		1·51428	
−Log 0·019938	=	−$\bar{2}$·29969	
	Log^{-1}	3·21459	= £1,639.

The total actual value of these mineral rights is, therefore, £81,622 + £1,639, or £83,261.

As before, the probable error in determining the quantity of mineral is 7½ per cent., and as it is essentially the quantity of coal that determines the value of the royalty, this same error may be taken as affecting the value as above determined, so that the value of the mineral rights may be set down as £83,261 ± 6,245;

it will at once be seen that the margin of probable error is very much less than was the case in the valuation of the mine, and this corresponds to the generally admitted fact that the purchase of mineral rights is far less of a speculative undertaking than the acquisition of a colliery where mining risks have to be reckoned with, and which involves considerable preliminary outlay before it can be brought into value, whereas mineral rights require no such preliminary expenditure.

The question may further arise what the value of these mineral rights would be if the purchaser could only undertake to get the mine working within five years from the date of starting work, but that this date would be itself deferred, assuming that this delay were due to some inherent quality of the mineral property, and not due to any deliberate action of the purchaser. If it were practically impossible to commence preliminary operations before five years from the date of the transaction; for example, if the locality at which the mine is to be opened up were not to be reasonably accessible until that time, as might be the case if a railway were under construction, but would not be open to traffic before the expiry of five years, the valuer might fairly consider that the property has no value as a mineral property until it is thus rendered accessible, and he would then have to take this additional five years of deferment into account. He would then probably calculate the value of the royalties by Formula 20:

$$\frac{4{,}465\ (1{\cdot}016^{34} - 1)}{0{\cdot}016 + 0{\cdot}0228\,(1{\cdot}016^{54} - 1)} = £67{,}892.$$

The dead rent would certainly be payable from the commencement of mining operations, and the question whether it should or should not be paid for the five years during which no work at all is possible would be

settled by agreement between the buyer and the seller. The seller would maintain that he is entitled to a consideration for parting with his property now, seeing that, if he waited until the railway were opened, he might expect a better price, whereas the buyer would point to the fact that the property had no productive value at all at the moment. Possibly the full royalty might be paid for the whole period preceding operations, or it is possible that the owner might accept a reduced royalty for the first five years; whichever of these alternatives is ultimately agreed on, the calculation presents no difficulties.

If, on the other hand, the property is ready to be worked as soon as it has been acquired, but the purchaser in order to suit his own purposes desires to leave it unworked for five years, the valuer will probably decide that he ought to pay for the right of so doing, and that the vendor should not receive any less than the price that would equitably be put upon it by a purchaser prepared to work it forthwith. This illustration affords an example of the differences as to the value of a mineral property that may well arise between buyer and seller. Apart from the above question of deferment, and apart also from the different views that the two parties may entertain as to what constitutes an equitable tonnage royalty, they may differ as to the rate of interest that may legitimately be expected to be earned by such an investment; the vendor would base his calculations upon the higher tonnage figure, and would regard 20 per cent. as too heavy a deduction, whilst the purchaser would take the lower tonnage figure, and might even want a deduction of 25 per cent. so as to keep him safe. In the above calculation colliery consumption has been allowed for in accordance with the usual practice, but here again the vendor might argue that he is entitled

to be paid for all the coal that can be worked out of his property, whilst the purchaser might fairly reply that he cannot be expected to pay for the coal that is consumed in getting out the remainder, and that, as he can only make his profit out of the latter, he should only pay for that quantity. These arguments are only adduced to show that, whilst the valuer's duty is to put as equitable a value as he possibly can upon the property, this figure will only form a basis for negotiations, so that the price ultimately paid, though it will bear some relation to it, will not as a rule be settled by it, although cases have occurred in which buyer and seller have agreed to accept the valuer's figure as an arbitral amount to be paid for a mineral property.

For instance, if the vendor were a wealthy man with an ample income derived from other sources, whilst the purchaser were the owner of an iron works which must have this coal supply or else close down, it is highly probable that the former would be able to hold out for the top price. If, on the other hand, the coal owner had just inherited the estate with heavy succession duties to pay, whilst the purchaser were a colliery company with ample coal reserves, the price agreed upon would probably be a much lower figure.

These considerations do not concern the valuer; his duty is to make neither a vendor's nor a purchaser's valuation, but to estimate what he considers to be the equitable value of the property such that the buyer and seller should both remain in practically the same pecuniary position after as before the transaction, whilst he indicates precisely the data upon which he bases his valuation, and then leaves it to his client to make as good a bargain as the circumstances admit of.

If for any reason the valuer holds that income tax should not be taken into account in determining the

present value of the property, the proposition will assume the following form:—The annual income is 47,000 tons at 6d., or £11,750; mineral rights duty at 1s. in the £ must be deducted, leaving an annual income of £11,162·5, or a half-yearly income of £5,581·25. The accumulative rate is, as before, 3·2 per cent. per annum, or 1·6 per cent. per half-year, because here income tax must necessarily be taken into account, seeing that it is deducted from the income derived from gilt-edged investments, in which the sum destined to the reimbursement of the original capital is assumed to accumulate. The risk rate of interest is $\frac{19}{20} \times 6$, or 5·7 per cent. per annum, equal to 2·85 per cent. per half-year. Deferment for five years is allowed as before, so that, in accordance with Formula 20,

$$P = \frac{5{,}581{\cdot}25 \times (1{\cdot}016^{34} - 1)}{0{\cdot}016 + 0{\cdot}0285\,(1{\cdot}016^{44} - 1)}$$

$$= \frac{5{,}581{\cdot}25 \times 0{\cdot}7155}{0{\cdot}016 + (0{\cdot}0285 \times 1{\cdot}0106)}.$$

$$\begin{aligned}
\text{Log } 1{\cdot}0106 &= 0{\cdot}00458 \\
\text{Log } 0{\cdot}0285 &= \bar{2}{\cdot}45484 \\
\text{Log}^{-1}\ \bar{2}{\cdot}45942 &= 0{\cdot}02880 \\
&\quad +0{\cdot}016 \\
&\quad 0{\cdot}04480 \\
\text{Log } 0{\cdot}7155 &= \bar{1}{\cdot}85461 \\
\text{Log } 5{,}581{\cdot}25 &= 3{\cdot}74673 \\
&\quad 3{\cdot}60134 \\
-\text{Log } 0{\cdot}0448 &= \bar{2}{\cdot}65128 \\
\text{Log}^{-1}\ 4{\cdot}95006 &= £89{,}138.
\end{aligned}$$

To this must be added the present value of the dead rent of £250 per half-year for ten half-years; deducting mineral rights duty, the income is £237·5. As before, the risk rate is taken as being the same for the dead rent as for the royalties—namely, 6 per cent. per annum —which, deducting mineral rights duty, is again equivalent to 2·85 per cent. per half-year. Again, using Formula No. 10, the actual value is found as follows :—

$$A = \frac{2{,}375\ (1{\cdot}016^{10} - 1)}{0{\cdot}016 + 0{\cdot}0285\ (1{\cdot}016^{10} - 1)}$$

$$= \frac{2{,}375 \times 0{\cdot}1720}{0{\cdot}016 + (0{\cdot}0285 \times 0{\cdot}1720)},$$

Log 0·1720	=	$\bar{1}$·23553	
Log 0·0285	=	$\bar{2}$·45484	
	Log^{-1}	$\bar{3}$·69037	= 0·004902
			+0·016
			0·020902
Log 0·1720	=	$\bar{1}$·23553	
Log 237·5	=	2·37566	
		1·61119	
−Log 0·0209	=	−$\bar{2}$·32015	
	Log^{-1}	3·29104	= £1,955.

The total value of the property would thus be

£89,138 + 1,955, or £91,093,

or about £7,800 more than was assigned to the property by taking income tax into account. It is, of course, evident that the purchaser will have to pay the income tax on his revenue in either case, but the latter valuation

takes no account of this fact. The difference is due to the fact that in the latter method of calculation a larger sum is supposed to be available for the formation of the sinking fund, as would be the case if income tax were payable only on true income. Taking the Formula for actual value (No. 20),

$$A = \frac{a}{s + \frac{i}{(1+i)^n - 1}} = \frac{a\,[(1+i)^n - 1]}{i + s\,[(1+i)^n - 1]},$$

in which a and s represent total revenue and total risk rate of interest without allowing for the deduction of income tax, it is admitted that the net accumulative rate must be employed, because investments can accumulate in no other manner, so that i represents the rate of accumulative interest after the deduction of income tax. Now, let the fraction representing the reduction of income due to income tax (in this case $\frac{16}{20}$) be c. Then allowing for income tax the above expression becomes :—

$$\frac{a\,c}{s\,c + \frac{i}{(1+i)^n - 1}} = \frac{a}{s + \frac{i}{c\,[(1+i)^n - 1]}}$$

$$= \frac{a\,[(1+i)^n - 1]}{\frac{i}{c} + s\,[(1+i)^n - 1]}.$$

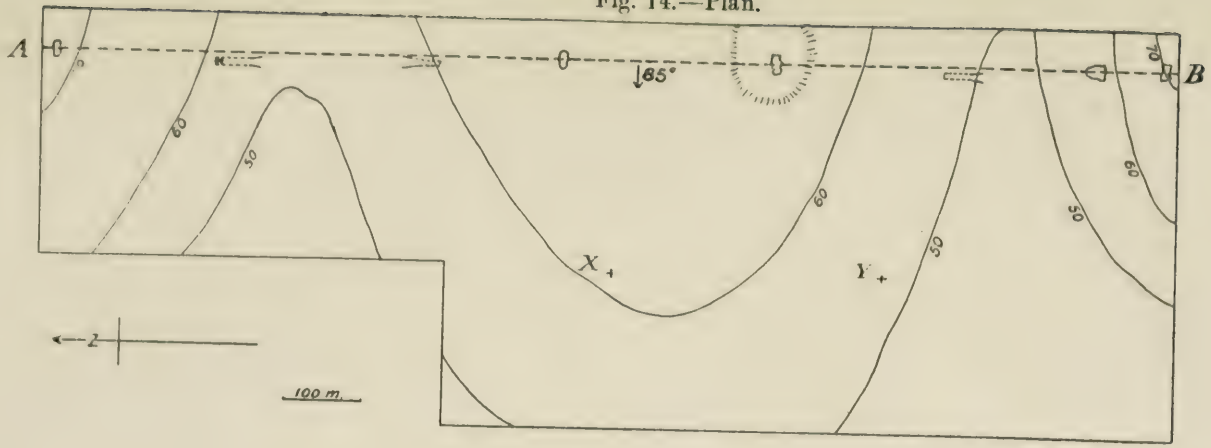
This naturally gives the same result as would be obtained by the use of Formula 20, in which a, s, and i were all taken as meaning the values obtained after deduction of income tax, and the Formula shows clearly why different results are obtained in the two calculations.

The question as to which of the two methods of valuation is the more correct can only be decided by a full consideration of all the circumstances as well as

the objects of the valuation. If, for example, the object of the valuation were to compare an investment in the purchase of the coal royalty with an investment in ordinary securities, from the revenue from which income tax would be deducted in the customary way, this latter method might be taken, because it compares revenues derivable under identical conditions. If, on the other hand, the object is to determine what price can be paid for the coal property so as to yield a definite net revenue, and make full provision for the replacement of capital, the first method is the one to be employed. This is one of the numerous instances that continually occur in valuations where the valuer has to use his judgment as to what the conditions of the case really imply, and where formulas used without intelligence will not suffice to lead to a correct conclusion. Generally speaking, the first method is the only correct one for ordinary British mineral valuations under the conditions at present existing in this country.

III. As another example, wholly imaginary again, let it be required to determine the value of a tin-bearing property in Spain held under concession from the Spanish Government, all the titles being proved in good order. Fig. 14 shows a sketch plan and Fig. 15 a vertical section of the property, which consists of 60 claims (*pertenencias*). A tin-bearing lode, indicated by the dotted line A B has been proved by the workings shown, and Fig. 15 is a section along the lode projected upon a vertical plane. There are indications of a second approximately parallel lode at points X and Y, but no work has been done upon it. Samples were cut at the points marked (1) to (11) on the section, and these have been assayed with the results shown below, the horizontal width of the vein having been measured at the same points :—

Fig. 14.—Plan.



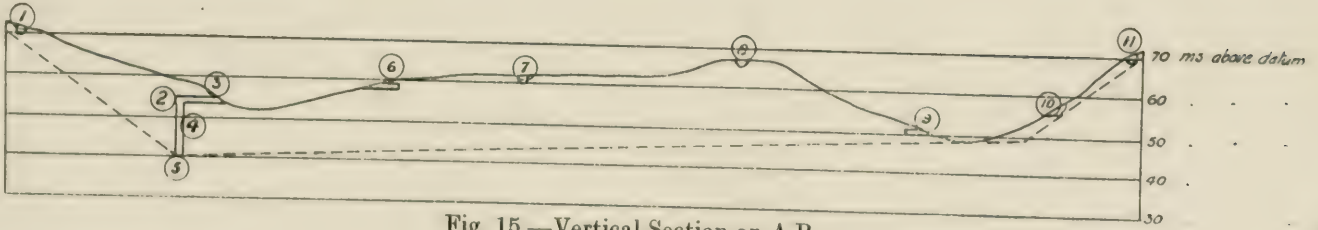


Fig. 15.—Vertical Section on A B.

Sample Nos.	Assays.	Horizontal Width.
	Per cent. of Tin.	Cm.
1, . . .	5·6	125
2, . . .	1·7	60
3, . . .	3·0	90
4, . . .	2·1	65
5, . . .	2·7	80
6, . . .	0·4	90
7, . . .	2·3	40
8, . . .	2·8	130
9 . . .	2·2	80
10, . . .	4·2	95
11, . . .	2·5	145

The average horizontal width of the vein is the arithmetical mean of the above series of measurements, or $\frac{1,000}{11} = 91$ cm. The probable error is obtained as shown in the following table :—

Widths.	Residuals (r).	r^2.
125 — 91	+ 34	1,156
60 — 91	— 31	961
90 — 91	— 1	1
65 — 91	— 26	676
80 — 91	— 11	121
90 — 91	— 1	1
40 — 91	— 51	2,601
130 — 91	+ 39	1,521
80 — 91	— 11	121
95 — 91	+ 4	16
145 — 91	+ 54	2,916
1,000		10,091

The probable error is, therefore, $0{\cdot}6745 \sqrt{\frac{10,091}{11 \times 10}}$ = 6·46 cm.

The average horizontal width is, therefore, put down as 91 ± 6·5 cm., the probable error being thus 7·2 per cent.

As regards assays, it is considered that twice as much weight should attach to the samples taken well below the outcrop, where the vein appeared to be practically unaffected by oxidation as to the outcrop samples, so that a weight (w) of 2 is given to samples 2, 3, 4, 5, and 9. The calculation is shown in the following table :—

Weighted Assays.	Assays.	Residuals (r).	Products ($w r^2$).
5·6	5·6 — 2·575	+ 3·025	9·150625
3·4	1·7 — 2·575	— 0·875	·765625
6·0	3·0 — 2·575	+ 0·425	·180625
4·2	2·1 — 2·575	— 0·475	·225625
5·4	2·7 — 2·575	+ 0·125	·015625
0·4	0·4 — 2·575	— 2·175	4·730625
2·3	2·3 — 2·575	— 0·275	0·075625
2·8	2·8 — 2·575	+ 0·225	0·050625
4·4	2·2 — 2·575	— 0·375	0·140625
4·2	4·2 — 2·575	+ 1·625	0·640625
2·5	2·5 — 2·575	+ 0·075	·005625
41·2			17·981875

The average assay is obtained by dividing the sum of the weighted assays by the sum of the weights—namely, $\frac{41{\cdot}2}{16} = 2{\cdot}575$ per cent. The probable error is given by the expression $0{\cdot}6745\sqrt{\frac{17{\cdot}981875}{10 \times 16}} = 0{\cdot}226$. The average percentage of tin is, therefore, 2·575 ± 0·226, the probable error amounting to 8·8 per cent.

In order to determine the quantity of mineral present, the area of the vein projected on a vertical plane must be multiplied by the average horizontal width. The

inspecting engineer finds that the vein extends into adjoining concessions, and that it shows every indication of permanency; he, therefore, considers that its persistence in depth down to the shaft bottom may be taken as fairly well proved—that is to say, that it is highly probable that the vein extends throughout the concession down to this depth; moreover, at the bottom of the shaft the vein was still holding well, there were no indications at all of a change of country, and no reason to consider that the vein was likely to pinch. The engineer, therefore, considers himself justified in assuming that it would continue below this point for a depth equal to half the height proved above it, or, say, for another 15 metres below the shaft bottom, but to this further quantity he only attaches half the weight that is given to the estimate of the portion above that point. The area down to the contour plane of 40 metres above datum (the level of the bottom of the shaft) can be taken out by planimeter, or by averaging the profile in the usual way. This will be found to be 23,800 square metres; taking the average horizontal width of 0·91 metre, this corresponds to a volume of 21,658 cubic metres. The volume to be reckoned on below the bottom of the little shaft is 21,000 cubic metres, but as only half the weight is given to this amount, it will be calculated as 10,500 cubic metres, making altogether a probable volume of 32,158 cubic metres. The vein stuff is found to have a specific gravity of 2·9, corresponding to a tonnage of 93,258 tons; these are metric tons, but in a calculation of this kind the difference between metric and statute tons (1 metric ton = 0·9842 statute ton) is generally negligible. The losses in working or due to cavities or other "wants" in a vein of this kind will not exceed 10 per cent., so that the quantity of workable ore may be taken as 83,932 tons, which

may reasonably be rounded off to 83,000 tons, this being a convenient way of making some allowance for the difference between metric and statute tons. Tin is at present worth about £160 per (statute) ton, and it may be assumed that it is not likely to fall below this figure for the period contemplated for the working of this mine. The inspecting engineer's tests warrant him in estimating that by crushing and dressing with proper machinery at least 85 per cent. of the tin will be recovered in the form of rough concentrates suitable for shipping, carrying 60 per cent. of metallic tin. The cost of mining is calculated at 10s. per ton, that of milling, transport, dues and taxes, shipping, management, etc., £1 per ton, calculated on the crude vein-stuff, so that the cost of production of concentrates is taken at 30s. per ton, to which must be added another 2s. per ton for development charges, or altogether 32s. per ton. The crude vein-stuff has been found to carry 2·575 per cent. of tin, or, on a recovery of 85 per cent., a yield of 2·189 per cent., contained in 3·648 per cent. of concentrates. The concentrates will, therefore, cost 877·2s. per ton delivered at the smelting works. With tin at £160 per ton, the concentrates will probably fetch £80 per ton, leaving a profit of 722·8s. per ton of concentrates, equal to a profit of 26·37s. or £1·3185 per ton of crude vein-stuff. It is assumed that the mine will take a year to open up and develop, including the erection of the necessary machinery, and the capital outlay required for this purpose is calculated to be £30,000. It is estimated that the mine will have a ten years' life, so that the annual profit is estimated at 8,300 tons at £1·3185, or £10,942·5.

It is considered that a reasonable rate of accumulative interest over the next eleven years is likely to be not far from $4\frac{1}{4}$ per cent., and that income tax may reasonably

be averaged over the same period at 4s. in the £, making the net rate of accumulative interest 3·4 per cent. It is held that for a mine of this kind the risk factor should be at least 3, so that a risk rate of interest of 12·75 per cent. may be required. The concession dues and the dues on tin ore payable to the Spanish Government and other authorities have been included in the cost of mining operations. The question now arises whether income tax is payable upon the whole of the profits of the mine or only upon the dividends distributed to shareholders, it being assumed that the company is a British one. There appears to be no doubt that if the profits are earned in this country, as, for instance, if the ore is shipped to this country and sold here, or if the whole of the profits are transmitted to this country, even although they are in fact earned abroad, as would be the case if the ore were sold abroad, the income tax would be payable upon the whole of the profits, as all legal decisions on the point support this view.*

If a portion of the profits is retained abroad, it apparently becomes a question of fact as to where the seat of control of the financial operations is situated. Thus, an English company was formed to work oil wells in Galicia, a portion of the profits was retained in Galicia for sinking new wells, and a portion remitted home for payment of dividends, etc. ; it was held that the company was taxable upon the whole of its profits.†

In the first cited case there was a local Board of

* See Egyptian Hotels, Limited, *v.* Mitchell, Surveyor of Taxes, King's Bench (No. 338), Nov. 25 to Dec. 2, 1912 ; De Beers Consolidated Mines, Limited, *v.* Howe, Surveyor of Taxes, High Court, April 12-17, 1905, Court of Appeal, June 1-6, 1905, House of Lords, June 18 to July 30, 1906.

† Elliots & Parkinson, Limited, *v.* Groves, Surveyor of Taxes, Queen's Bench (No. 189), May 13, 1896.

Directors in Egypt with full administrative functions, but the financial control was retained in this country. Apparently, in the special case under discussion here, if the Board of Directors consisted of Englishmen who met in Spain and transacted all the business of the company there, and merely transmitted such portion of the profits as they proposed to pay out in dividends to an agent at home, who would distribute them to the shareholders entitled thereto, income tax would only be paid on the portion of the profits so transmitted. Whether the purchase abroad of British securities to form a sinking fund and their transmission to this country would or would not render these too liable to income tax is perhaps doubtful, but it would appear probable that if both purchased and deposited abroad the money used in this way would probably escape taxation.

It will be assumed for the purposes of calculation that it is proposed to adopt a method having the above result, although, as already pointed out, the law on the matter is far from clear. It would appear that the mere fact that a company is registered in this country is of secondary importance, the determining factor being the question whether the seat of effective financial as distinguished from technical control is situated abroad or in this country. The owners of the mine will necessarily have to pay income tax on their dividends either individually or else by deduction before the dividend warrants are distributed, but this point does not affect the valuation, though it must be clearly understood that the risk rate of interest above given does not provide for it. The basis of valuation is accordingly that income tax on the dividends is not provided for in the valuation, and that it is assumed that no income tax is payable in respect of the sums set aside to form a reserve fund,

but that these sums are invested in British securities, the dividends from which are liable to income tax.

Using Formula 20, the present value will be given by the expression :—

$$\frac{10{\cdot}942{\cdot}5\,(1{\cdot}034^{10}-1)}{0{\cdot}034+0{\cdot}1275\,(1{\cdot}034^{11}-1)}.$$

The calculation is then as follows :—

Log 1·034 = 0·0145205
11
Log⁻¹ 0·15973 = 1·4445
−1
0·4445

Log 0·4445 = $\bar{1}$·64787
Log 0·1275 $\bar{1}$·10551
Log⁻¹ $\bar{2}$·75338 = 0·05667
+0·034
0·09067

Log 1·034 = 0·0145205
10
Log⁻¹ 0·14521 = 1·397
−1
0·397

Log 0·397 = $\bar{1}$·59879
Log 10942·5 = 4·03912
3·63791
−Log 0·09067 = $\bar{2}$·95746
Log⁻¹ 4·68045 = £47,912.

If, however, the constitution of the company or the

method of conducting business or any other reason should preclude the possibility of disposing of the profits as above, so that income tax is payable in respect of the whole of the gross profit of the mine, other conditions remaining the same, the profit available for distribution, allowing the above rate of income tax, will be

$$\frac{16}{20} \times £10{,}942{\cdot}5 = £8{,}754.$$

The risk rate of interest after deducting income tax will become

$$\frac{16}{20} \times 12{\cdot}75 = 10{\cdot}2 \text{ per cent.}$$

The formula, therefore, becomes :—

$$\frac{8{,}754\ (1{\cdot}034^{10} - 1)}{0{\cdot}034 + 0{\cdot}102\ (1{\cdot}034^{11} - 1)}$$
$$= £43{,}815.$$

In other words, the British method of charging income tax upon that portion of the profits, which is not in fact revenue, but redemption of capital, diminishes the original value of this proposition by more than £4,000, or over $8\frac{1}{2}$ per cent.

The above figure gives the total value of the mine at the commencement of the preliminary operations; to bring it to this value, however, it is necessary to spend £30,000 upon it. This sum is to be spent in the first twelvemonth, and it may reasonably be conceived that the expenditure will be spread uniformly throughout the year, to be paid, say, by monthly payments or remittances; its present value, therefore, taking the same rate of accumulative interest as above, will be given in accordance with Formula 4 by the expression

$$\frac{2{,}500}{0{\cdot}00283}\ (1 - 1{\cdot}00283^{-12}).$$

The calculation is as follows :—

Log	1·00283			
	−12			
Log^{-1}	−0·01473	= Log^{-1}	$\bar{1}$·98527	= 0·96665
	1 −		0·96665	= 0·03335
Log	0·03335	=	$\bar{2}$·52310	
Log	2500	=	3·39794	
			1·92104	
−Log	0·00283	=	− $\bar{3}$·45179	
		Log^{-1}	4·46925	= £29,461.

Hence the difference between these two amounts, £47,912 and £29,461, or £18,451, represents the sum that can fairly be paid for the mining concession. It will be noted that it is assumed that the plant will be practically worthless when the mine is worked out, and this is usually a prudent assumption to make when a mine is situated in a somewhat remote and difficultly accessible district. It has been seen that the probable error in estimating the average width of the vein has been found to be 7·2 per cent., and that of the average assay contents 8·8 per cent. The probable error in estimating the average value, which varies as the product of these two items, is $\sqrt{7{\cdot}2^2 + 8{\cdot}8^2}$, or 11·37 per cent.; 11·37 per cent. of the total value of the mine as a going concern is £5,447, so that the valuer is justified in estimating the present value of the property at £18,451 ± £5,447, or between £13,004 and £23,898.

There are various points in connection with this valuation which require consideration. It will be noticed that there is practically no ore blocked out at all in the proper sense of the word, and that the probability of its existence depends largely upon geological inference,

so that a purchaser accepting a valuation as above does so largely upon his faith in the ability and experience of the mining engineer. Some engineers would admit far less ore than has here been taken into calculation; they might say that no ore below the dotted lines shown on the section is practically proved, and would, therefore, allow only 13,000 square metres of proved vein, corresponding to a tonnage of approximately 34,300 tons. It has already been pointed out that if the inspecting engineer's geology is to be trusted, this must be far too conservative a figure on which to base a valuation, because the probability in favour of the ore extending at least to the depth above assumed is a very strong one; nevertheless, this figure of 34,300 tons of vein-stuff practically proved has an important use; if this ore is capable of realising a profit as above indicated of £1·3185 per ton, it should produce a total profit of £45,225, or at the worst it would thus be capable of practically returning the capital outlay upon the property, seeing that the capital outlay would necessarily be smaller if only the smaller body of ore is to be worked than it would need to be for treating the larger quantity. It is always a wise precaution to take care that this minimum requirement, namely, that the ore practically proved shall be sufficient to repay the expected outlay, is highly probable of realisation, so that, at the very worst, the mining speculators would get their capital back. It is even a question whether an engineer is justified in recommending the purchase of a mining property unless he is well satisfied that this minimum condition will be fulfilled.

It has been shown that there are indications of the existence of another vein at X Y, apparently parallel to the first one, on which no work at all has been done. A proper valuation of these indications is, of course,

out of the question ; it may be admitted that the property is worth more with these indications than it would be without them, but the increase in value is very difficult to quantify. Many engineers might be inclined to neglect it altogether, but this would appear to be inequitable. It might be suggested that a knowledge of the district might be turned to account. If, for example, it were known that in the surrounding districts there were a couple of hundred of such indications which had been tested, and that in four instances profitable mines had been successfully opened upon them, it might be argued that the probability of this prospect turning out to be a payable vein is 1/50th. It will be seen from the plan that its length within the concession, if it does turn out to be a true vein, can only be 900 metres as against 1,400 metres upon the first vein, so that, if the two veins were equal in all other respects, the value of the shorter one would be 9/14ths of the longer one, or about £30,000, assuming that no additional capital is required to work it. If the probability of realising such a sum is 1/50th, the value that may be assigned to the above indications is £600. It will be seen that this is a very speculative and rough method of estimating the value of such an indication, but, in default of anything better, such methods have at times to be employed when it is absolutely necessary to put a money value upon it.

It is quite evident that the whole proposition still rests upon a somewhat speculative basis ; obviously the proper course would be, if possible, to arrange with the owner of the property for an option of six to twelve months' duration, in order to enable the prospective buyer to test the property thoroughly before concluding his purchase. It may here be pointed out that owners of such properties often ask for a money payment,

sometimes quite an important one, for the grant of such an option; in the writer's opinion there is no justification for such a demand. If the prospective purchaser is prepared to spend money in proving the property, he is doing work that should by rights have been done before by and at the expense of the owner of the property, and the latter is not warranted in asking to be indemnified for his own default. It is equitable that the would-be purchaser should undertake to spend a definite amount, to be mutually agreed upon, in testing the property, and that he should pay all the taxes and dues payable on the concession during his period of occupation, but nothing more. The vendor receives a consideration in return for his option, inasmuch as he gets his property tested for him for nothing, and he usually stipulates that, if the option is not taken up, he shall receive full information as to the results of the testing operations. If, at the expiration of the option time, the prospective purchaser has not yet made up his mind concerning the property, and wishes an extension of the option, then the vendor might perhaps fairly require him to pay for such a prolongation of the option, but it would not appear to be unjust that the prospective buyer should pay a penalty for having taken too short an option period in the first instance. It is often arranged that, if the option holder takes up the option, the money expended by him in testing the property shall form a deduction from the purchase price. Even when the proper course of taking an option and thoroughly testing the property is followed, a valuation such as the above becomes necessary, because the option agreement must set forth the price at which the vendor is prepared to sell, and the reasonableness or otherwise of such price can only be ascertained by such a method of valuation. Necessarily the price becomes a maximum, being the

figure at which the owner is bound to sell if the option-holder exercises his option; nothing, however, prevents the latter from declining to purchase at the option price and offering a lower figure, which perhaps his exploratory work has shown him is as much as he can afford to pay for the property; the price to be ultimately paid then becomes a matter of bargain between the parties, as in all such cases, the valuation affording only the starting point for the discussion.

IV. Assume that a mine has been opened on a vein of auriferous quartz, the vein being nearly vertical, and that the mine has been worked for some little time. A vertical shaft has been sunk down to a depth of 550 feet in country rock (mica schist), which runs uniformly all the way down. A shallow drift has been put in at 20 feet below the shaft collar, the latter being near the bottom of a small valley, and the vein has been worked out above this level for the full length of the pay shoot, which measures 800 feet along the vein, and has produced 11,200 tons of ore. Levels have been set off at 100 feet intervals all down the shaft; the 100 feet level—*i.e.*, the level 100 feet below the shallow level—has been driven 600 feet in the pay shoot and the ends are still in good ore. The 200 feet level has been driven 200 feet, and the 300 feet level 50 feet, also all in good ore, and the 400 and 500 feet levels have cut the vein and have been driven a few feet on it to test its character; the vein has also been touched from the bottom of the sump at 550 feet.

The oxidised zone of ore terminates above the shallow level, so that practically the whole of that level is in ore in its unaltered or very slightly altered condition. The vein is in a gold mining district, similar pay shoots have been worked on the continuation of the vein to a considerably greater depth than has been reached in

the present case, and other similar veins have held down in the same way without showing any marked decrease either in width of vein or in length of pay shoots or in the value of the ore. The mine is fully equipped, and can treat 5,000 tons per annum or possibly a little more. It has been in full operation for three years, and has treated 15,000 tons ; the bulk of this has come from above the shallow level, which may be looked upon as practically worked out, whilst 3,800 tons have been stoped above the 100 feet level. The results of working and sampling are shown in the following table :—

	Assays.	Average Assay of Ore.	Average Horizontal Width.
Shallow level, . .	12-33 dwts.	(stoped) 21 dwts.	3 ft.
100 ft. level, . .	8-14 ,,	(stoped) 10 ,,	2 ft. 9 in.
200 ft. level, . .	7-13 ,,	9½ ,,	2 ft. 11 in.
300 ft. level, . .	6-13 ,,	9 ,,	2 ft. 8 in.
400 ft. level, . .	..	6 ,,	3 ft. 3 in.
500 ft. level, . .	..	8 ,,	2 ft. 7 in.
Sump, . . .	..	10 ,,	2 ft. 10 in.

The cost of mining and milling the unoxidised ore inclusive of everything has been found to be 27s. per ton. There are no special difficulties, and the mine makes but little water.

An offer to purchase the mine has been received from a syndicate controlling a number of adjoining properties, and a valuation of the mine is accordingly required. Large profits were made in working the shallow ore, and the undertaking so far has been a remunerative one, though, of course, the property is only small. No doubt the amalgamation will result in certain economies in administration and manage-

ment, and may also diminish milling costs, but with this the valuer has nothing to do. Neither is he concerned with the expenditure that has taken place up to the present on the property; it makes no difference how much the mine cost originally, or how much money has been spent upon development, plant, and equipment; money may have been wasted on the latter, or the expenditure may have been thoroughly judicious; neither of these alternatives affect the present value of the mine, which must be determined purely upon the basis of a going concern as it exists to-day.

In order to determine the average assay value and horizontal width, the data given above must be utilised. Obviously there is more information concerning the first three levels than there is concerning the two latter and the sump, and the observations are weighted accordingly, a weight of unity being attached to the three last figures, a weight of 2 to the observations and averages on the 300 feet level, a weight of 4 to those on the 200 feet level, and a weight of 8 to those on the 100 feet level. The following table shows the calculation:—

Weights (w).	Values (v).	Product (vw).
8	10 dwts.	80
4	9·5 ,,	38
2	9 ,,	18
1	6 ,,	6
1	8 ,,	8
1	10 ,,	10
17		160

The average value (m) is, therefore, $\frac{160}{17} = 9{\cdot}4$ dwts.

The probable error of the above average is worked out as follows :—

$v - m$.	r.	r^2.	$w r^2$.
10 — 9·4	+ 0·6	0·36	2·88
9·5 — 9·4	+ 0·1	0·1	0·40
9 — 9·4	— 0·4	0·16	0·32
6 — 9·4	— 3·4	11·56	11·56
8 — 9·4	— 1·4	1·96	1·96
10 — 9·4	+ 0·6	0·36	0·36
			17·48

The probable error, therefore, is

$$0{\cdot}6745 \sqrt{\frac{17{\cdot}48}{5 \times 17}} = 0{\cdot}31.$$

The average value may, therefore, be set down as $9{\cdot}4 \pm 0{\cdot}31$ dwt., corresponding to an error of 3·3 per cent. For the widths the calculation is performed on similar lines :—

Weight (w).	Widths (t).	Products (wt).
8	33 inches	264
4	35 ,,	140
2	32 ,,	64
1	39 ,,	39
1	31 ,,	31
1	34 ,,	34
17		572

The average horizontal width is, therefore, $\frac{572}{17} = 33{\cdot}6$ inches = 2 feet 9·6 inches, or 2·8 feet.

The following is the calculation for the probable error :—

$t - m$.	r.	r^2.	$w r^2$.
33 — 33·6	— 0·6	0·36	2·88
35 — 33·6	+ 1·4	1·96	7·84
32 — 33·6	— 1·6	2·56	5·12
39 — 33·6	+ 5·4	29·16	29·16
31 — 33·6	— 2·6	6·76	6·76
34 — 33·6	+ 0·4	0·16	0·16
			51·92

The probable error of the above average, therefore, is $0{\cdot}6745 \sqrt{\frac{51{\cdot}92}{5 \times 17}} = 0{\cdot}52$. The average width may, therefore, be put down as 2 feet 9·6 inches $\pm$ 0·5 inch. The probable error is, therefore, 1·6 per cent. The combined error is, therefore, $\sqrt{3{\cdot}3^2 + 1{\cdot}6^2}$ per cent. = 3·67 per cent.

As regards the quantity of ore, the amount actually proved may not be very great, but there appear to be good reasons to expect that the pay shoot will continue to extend to the full length of 800 feet in the lower levels. The vein holds well, and is apparently as rich as ever at 550 feet, and there is no reason to doubt that it will go down even below this point. The valuer, however, does not feel justified in calculating upon this hypothesis, but assumes that the shoot will be at least 200 feet long at a depth of 550 feet, and that it will lengthen uniformly up to 800 feet on the shallow level. This corresponds to an area of 275,000 square feet ; he considers that the vein may be looked upon as holding down to a depth of 100 feet below the bottom of the sump, and that it tapers from 200 feet to nothing, and, therefore, allows an additional 10,000 square feet for

this portion, making a total of 285,000 square feet. At an average thickness of 2·8 feet, this corresponds to a volume of 798,000 cubic feet. It has been found by actual test that this ore goes 14·7 cubic feet to the ton, so that the total tonnage thus admitted is 54,300 tons below the shallow level; out of this 3,800 tons have already been stoped, leaving a little over 50,000 tons of ore to be won, or ten years' supply.

The average richness has been found to be 9·4 dwts. per ton. Taking the probable value of gold over the life of the mine at £4 12s. per ounce, and considering that an extraction of 93 per cent. has been regularly obtained on the unoxidised ore, it will be seen that the gold produced may be taken at 0·437 ounce, worth 40·2 shillings, per ton. The cost of treatment, everything included, has been found to be 27s. per ton of ore. It is reasonable to expect that it may be a little greater in depth, and may, therefore, be put at 30s., leaving a profit of 10·2s. per ton, or on 5,000 tons £2,550 per annum.

The accumulative rate of interest will be taken at 4 per cent. less income tax, assumed at 4s. in the £, or a net rate of 3·2 per cent. Having regard to the conservative estimate of quantity, a reasonable risk factor would be 2·5; the present owners have taken the greater part of the mining risk when they invested their money in what was originally a prospect, and they are entitled to the reward that attends a successful venture. Their work has gone so far towards proving the mine, especially taking into consideration the result of adjoining properties, that high risks may be looked upon to a great extent eliminated, so that a risk rate of 10 per cent. may be looked upon as equitable. The question of income tax does not arise as far as profits are concerned. The mine is a foreign one, and may be owned by foreigners who are not accountable for income

tax to this country, or, if in British hands, these might choose to leave their profits abroad, and thus escape the payment of income tax.

Let it be assumed that dividends are payable quarterly, whilst the accumulative interest is payable half-yearly. Formula 11 will, therefore, be applied, and the actual value is given by the following expression :—

$$\frac{2{,}550\ (1{\cdot}016^{20} - 1)}{4\ (1{\cdot}016^{\frac{1}{2}} - 1) + 0{\cdot}1\ (1{\cdot}016^{20} - 1)}.$$

The calculation is, therefore, as follows :—

Log 1·016 = 0·0068937
20
Log^{-1} 0·1378740 = 1·3736
−1
0·3736

Log 1·016 = 0·0068937
·5
Log^{-1} 0·0034469 = 1·00796
−1
0·00796
4
0·03184
+ 0·03736
0·06920

Log 2550 = 3·40654
Log 0·3736 = $\bar{1}$·57241
2·97895
−Log 0·0692 = −$\bar{2}$·84011
Log^{-1} 4·13884 = £13,767.

It may be interesting to work out the actual value if both dividends and accumulative interest are payable half-yearly. In this case Formula 12 must be used, and the actual value is given by the expression :—

$$\frac{2{,}550\ (1{\cdot}016^{20} - 1)}{0{\cdot}032 + 0{\cdot}1\ (1{\cdot}016^{20} - 1)}$$

$$= \frac{2{,}550 \times 0{\cdot}3736}{0{\cdot}032 + 0{\cdot}03736}$$

$$= \frac{2{,}550 \times 0{\cdot}3736}{0{\cdot}06936}.$$

The calculation is then as follows :—

$$\begin{aligned} \text{Log } 2550 &= 3{\cdot}40654 \\ \text{Log } 0{\cdot}3736 &= \bar{1}{\cdot}57241 \\ & \qquad\qquad 2{\cdot}97895 \\ -\text{Log } 0{\cdot}06936 &= \bar{2}{\cdot}84111 \\ & \qquad\qquad 4{\cdot}13784 = £13{,}735. \end{aligned}$$

If both dividends and accumulative interest are payable yearly, Formula 10 would give as the expression for the present value :—

$$\frac{2{,}550\ (1{\cdot}032^{10} - 1)}{0{\cdot}032 + 0{\cdot}1\ (1{\cdot}032^{10} - 1)}.$$

The following is the calculation :—

$$\begin{aligned}
\text{Log } 1{\cdot}032 &= 0{\cdot}0136797 \\
&\quad\ 10 \\
\text{Log}^{-1}\ 0{\cdot}136797 &= 1{\cdot}37025 \\
&\quad -1 \\
&= 0{\cdot}37025 \\
&\quad\ 0{\cdot}1 \\
&= 0{\cdot}037025 \\
&\quad + 0{\cdot}032 \\
&= 0{\cdot}069025 \\
\text{Log } 2550 &= 3{\cdot}40654 \\
\text{Log } 0{\cdot}37025 &= \bar{1}{\cdot}56849 \\
&= 2{\cdot}97503 \\
-\text{Log } 0{\cdot}069025 &= \bar{2}{\cdot}83900 \\
\text{Log}^{-1}\ 4{\cdot}13603 &= £13{,}678.
\end{aligned}$$

In addition to the figure (£13,767) which represents the present value of the standing ore, a certain amount of development work has been done, which will cheapen the cost of that ore, and this should be taken into account in the valuation. 850 feet of level have been driven, developing approximately 16,000 tons of ore, of which 3,800 tons have been stoped, leaving an amount of something over 12,000 tons on which drivage development has been done ; the value of the ore extracted in the drivage has gone some way to recoup the cost of this, and 2s. per ton may be considered a fair allowance

for the cost of this development work, or, say, £1,200. The ore thus developed may not—probably will not—all be worked forthwith, but it is ready for working, and it would seem to be equitable to add this sum to the value as found above. It has been seen that the probable error in respect of assay value and thickness of vein is only 3·67 per cent., so that the valuation arrived at would be

$$£13{,}735 + £1{,}200, \text{ or } £14{,}935 \pm 504.$$

It will have been seen, however, that in this case the greatest measure of uncertainty attends the estimate of the quantity of ore available. Adjoining mines have shown that these ore shoots hold well in depth, and the possibility that the pay ore continues to its full length of 800 feet down to a depth of at least 650 feet cannot be ignored. There is at any rate a decided degree of probability that such will be the case, although that probability cannot be quantified. If the ore shoot carried down in full length to that depth, the tonnage on the same data as previously taken would be about 99,000 tons, of which 3,800 tons have been taken out, leaving 95,200 tons, or a 19 years' supply.

In order to allow for the greater risk undoubtedly presented by this hypothesis, a larger risk factor will have to be adopted, say $3\frac{1}{2}$ instead of $2\frac{1}{2}$, or a risk rate of interest of 14 per cent. per annum. Completing the calculation on this basis, and for the sake of simplicity assuming that both the dividends and the accumulative interest are payable yearly, the valuation formula No. 10 would result in the expression :—

$$\frac{2{,}550\ (1{\cdot}032^{19} - 1)}{0{\cdot}032 + 0{\cdot}14\ (1{\cdot}032^{19} - 1)}.$$

The calculation will be as follows :—

Log 1·032	= 0·0136797		
	19		
	Log^{-1} 0·2599143	=	1·81975
			−1
			0·81975
			0·14
			0·114765
			+ 0·032
			0·146765
Log 0·81975	= $\bar{1}$·91368		
Log 2550	= 3·40654		
	3·32022		
−Log 0·146765	= −$\bar{1}$·16662		
	Log^{-1} 4·15360	=	£14,243.

It will be seen that on these lines the value is £565 more than was obtained by the more conservative method of calculation in spite of the fact that a much higher rate of interest has been allowed. This consideration indicates that, whereas the first method of calculation is probably the one that would be followed by a valuer, the purchaser has at any rate a very good chance of doing well on the bargain.

Here, however, another consideration comes into play. If there is a strong enough probability to warrant the assumption of the existence of so relatively important a tonnage, or if such in fact exists, there is no reason why the plant should not be increased so as to admit of raising and treating 10,000 tons per annum. This would in a case like the present probably mean a slight

increase in winding power or a modification of the winding appliances and an increase in rock-drilling plant, together with additional equipment, such as mine trams, rails, etc., but the main cost would be in the additional plant for ore treatment. An increase of 5,000 tons per annum means an increase in milling capacity of about 16 tons per working day; the cost will vary between very wide limits according to the methods of gold extraction applicable to the case; it might according to circumstances range from £50 to £200 per ton per day. Let it be assumed that in the present instance the total additional cost will be £2,000. The effect, no doubt, would be to cut down working costs, say by 2s. per ton, so that a profit of 12·2s. per ton might now be expected, or on 10,000 tons £6,100 per annum, the life of the mine being reduced to 9·5 years. All other conditions being taken the same as in the last case, the actual value will be represented by the expression :—

$$\frac{6{,}100\ (1{\cdot}032^{9{\cdot}5} - 1)}{1{\cdot}032 + 0{\cdot}14\ (1{\cdot}032^{9{\cdot}5} - 1)}.$$

Performing the calculation exactly as before, the value will be found to be £26,322. From this must be subtracted the additional capital rendered necessary —namely, £2,000—so that the corresponding value of the property under these conditions becomes £24,322, an increase of something like 70 per cent. It should be noted that in all cases the value of the plant has been put down as nil when the mine is worked out.

Attention must be directed to the increase thus shown in the value of a property due to speeding up the rate of working. It has already been pointed out that this increase cannot continue indefinitely, but that there are economic limits, any increase beyond which involves a falling off in the total profits obtainable.

Within these limits, however, an increase in the rate of realisation of the values locked up underground causes a marked increase in actual value, particularly when the risk factor is a high one. In this connection the interesting point arises, to what extent working out the richest part of the property first in order to realise its value most promptly may or may not be a sound policy; as a general rule, the practice of "picking the eyes out of a mine" is a reprehensible one, especially when it causes large portions of mineral to be left behind as unworkable, whereas they could have been worked so as to yield a profit if a due proportion of the richer mineral had been worked with them. Ethically it can no doubt be argued that it is the duty of the mine worker to extract the largest amount of mineral that can be extracted for the use of mankind, but in practice mines will naturally always be so worked as to yield the maximum possible profit to the owners; the determination of the conditions that will produce this maximum profit presents no little difficulty. Thus, it has been shown that intensive working produces a high present value when a high rate of interest is involved, but it must be remembered that this high rate is the mine speculator's reward for a successful speculation; it does not follow at all that he can obtain any such rate of interest for the revenue which he gets out of the mine, and this has value for him as a source of revenue only at the ordinary accumulative rate. It is thus quite conceivable that in times when the rate of interest on gilt-edged securities is low, a slower rate than the maximum of economic intensity might prove to be the most profitable —that is to say, will have produced the largest sum of money in mine profits plus the accumulative interest on these profits at the end of the life of the mine. It has been only possible here to indicate the complexity of

the problem, and to show how various are the considerations that must be taken into account when deciding what is the most economic rate of working any given mine; each case must be investigated individually. The bearing of this aspect upon the question what price a speculator can afford to pay for a given mine can to some extent be inferred from what has been said, whilst it also becomes clear that such price may vary within certain limits according to the circumstances of the purchaser apart from the intrinsic value of the property, though here again this consideration may be held to lie outside the province of the valuer.

V. A company owning an iron works has an opportunity of acquiring the fee simple of an estate of about 700 acres, of which about 560 acres are underlain by a bed of ironstone in a well-known ironstone field. There are operating iron mining companies at work on either side of this estate, and the iron works have been in the habit of buying their ore supply from one or other of these, the quality of the ore being practically the same in both cases. The thickness of the bed of ironstone in both the working properties has averaged 7 feet 6 inches, varying from 6 feet 9 inches to 9 feet. The ironstone bed outcrops in the property and can be examined along its outcrop, where its thickness and composition correspond to the bed worked in the adjoining properties. Some fifteen boreholes have been put down, and have everywhere proved the ironstone bed to be within the limits of thickness above stated, the average working out at 7 feet 8 inches $\pm$ 4·6 inches, equivalent to a probable error of 5 per cent., and the analyses of the cores are very similar to those of the ore being regularly received from the adjoining properties. The geological structure of the entire field is very uniform.

It is, therefore, concluded that it is safe to reckon upon a bed of ironstone 7 feet 6 inches thick under the whole of the iron-bearing area of 560 acres, and that its composition and, therefore, its value will be practically identical with that of the ore with which the iron works have been familiar for a number of years. The consumption of ore by the works is 500,000 tons per annum.

If the iron works do not purchase this property, there is little doubt but that one or other of the ore producers on either side will do so, and the iron works would then be compelled to buy their ore supplies in the open market as heretofore. The iron works hope, not only to make a fair profit on the purchase, but to secure the further advantage that they will control their own supply of raw material and be independent of market fluctuations and free from the danger of any adverse combination of ore producers, whose object would be to unduly force up the price of ironstone. Such advantages are not indeed quantifiable, but they are very real, and have before now been held to justify the expenditure of considerable capital, even when the probability of making a direct profit on the purchase was somewhat remote. It is known that a large portion of the ironstone bed lies under very shallow cover, and can be worked open-cast, using mechanical shovels (steam or electric) for stripping off the over-burden and for getting the ore. It is estimated that approximately half the ore can be worked open-cast and half by underground mining; as the depth of over-burden gradually increases, the advantages of open-cast over underground mining gradually get less, until a thickness of over-burden is reached at which underground mining becomes the more economical. This critical thickness depends upon many conditions, and cannot be predicted with

any confidence some years ahead, so that the ratio of the ore mineable open-cast to ore mineable underground can only be stated approximately; moreover, in practice it is sure to happen that during a certain period of time a portion of the output will be got by both these methods. For the purposes of calculation a clean cut is, however, assumed, and the period of over-lapping is disregarded. For a while, at any rate, open-cast mining will certainly be more profitable than underground mining can be. The iron works Company, which is a prosperous concern in good standing, decides to raise the money which it will need for the purchase and equipment of this ore field by means of a debenture issue, and it proposes to redeem these debentures during the first ten years of the life of the mine. It is required to determine what sum of money the iron works can afford to pay for the freehold under these conditions, and what the probable result of the investment will be.

It is proper to point out that the purchase will involve the acquisition of the surface of the land as well as of the minerals. As the valuation of the former does not enter into the scope of this work, it will be assumed that the iron works purchase the land out of a separate fund, which is administered quite apart from the purchase of the minerals. Estate valuers will determine the value of the surface; probably it will consist of farms, all of which are let, so that the annual income obtained therefrom is readily ascertained. Care is taken to see that the fields, farm buildings, etc., are in reasonably good condition, and that the annual income derivable therefrom can accordingly be maintained. The actual value of the surface will, therefore, be the value of a perpetuity corresponding to the income from the rent of the land less all necessary outgoings.

A problem that here presents itself is whether it is right to value the surface at its present full rental value, having regard to the fact that this value will be diminished and may even in part be practically destroyed by the mining operations; this is a question which is answered in different ways by different authorities. Upon the whole, it would seem to be sound practice to look upon such diminution in value, if temporary, as a decrease in rental value which must be made good by a corresponding charge on the ore extracted and, if permanent, by a similar charge sufficient to make up the actual value of a perpetuity equal to the loss of rental; this must also be borne by the ore extracted, and in either case they are best charged upon a tonnage basis. It is self-evident that if the net profit on the ore extracted is less than these charges for surface damage the ore will be left standing, otherwise the ore will be worked and the surface damage will constitute an item in the cost of ore production.

In order to determine the probable ore contents, it will be wiser to assume the average thickness as equal to that found in working the adjoining portions—namely, 7 feet 6 inches, though this is somewhat less than the figure obtained by boring. One acre of water 1 foot thick equals 1,215 tons; the specific gravity of the oxidised ore under shallow cover has been found to be 2·1, and that of the deeper lying unoxidised ore 2·2, making the respective tonnages per foot thick per acre 2,552 tons and 2,673 tons. At a thickness of 7 feet 6 inches these figures give 19,140 and 20,048 tons per acre; allowing 8 per cent. and 10 per cent. loss in working respectively, the produce per acre will be calculated at 17,609 and 18,043 tons per acre respectively. The acreage underlain by ore being 560 acres, of which one-half may be taken as underlain by shallow ore and the other half

by deep ore, the quantity of available ore is thus calculated at about 10 million tons, and 5 million tons will be taken as being workable by either method—that is to say, the quantity of ore is reckoned as equal to a ten years' supply of ore worked open-cast, which will be won first, to be followed by a further ten years of ore supply workable by underground mining.

The royalties paid in the district average 7½d. per ton for ore workable open-cast and 4½d. per ton for ore workable by underground mining; as there is a natural tendency for all such royalties to rise in proportion as the ironstone in the field approaches exhaustion, it is fair to increase these figures slightly and to call them 8d. and 5d. respectively. The annual income for the first decade will, therefore, be 500,000 × 8d., or £16,666·65, and for the second decade 500,000 × 5d., or £10,416·65. Mineral rights duty must, however, be deducted from these figures, reducing them to £15,833·32 and £9,895·82 respectively. No question of income tax arises, because any profits due to the mining investment will be treated as part of the profits of the iron works. As a matter of book-keeping, it is usual and in many respects desirable that the mining proposition should be treated in the accounts as though it were an independent venture, and be credited with the value of the ironstone produced at market price, so that it may be seen whether the mining venture is in fact a successful one or not, but this is purely a question of internal administration, and, as far as the income tax authorities are concerned, any profits ascribable to the mining operations will be treated as profits made by the iron works.

For the gross accumulative rate of interest, 4 per cent. is taken as a fair average figure; it is estimated that income tax for the first decade is likely to average 4s. in the £, and for the second period of ten years 3s.

in the £, so that the net accumulative rates for these two periods are taken as 3·2 and 3·4 per cent. respectively. A risk factor of 2 is taken as fairly representing the situation, and the investment is calculated on the basis of its yielding 8 per cent. net—that is to say, after allowing for the payment of mineral rights duty—corresponding, therefore, strictly speaking, to a somewhat higher rate of interest, the iron works having stipulated that this is the lowest return on capital that they are prepared to consider.

This is practically equivalent to saying that mineral rights duty is looked upon as an item in the cost of getting the ore, which is, in fact, quite a reasonable view, and one that is adopted by many accountants in preparing statements of the cost of ore production.

Royalties and interest are both taken as payable half-yearly. As the ore workable open-cast can be attacked forthwith, or at any rate in the course of a very few months, no deferment will be allowed in the valuation, and the actual value of the royalties for the first decade will be determined by Formula 12, giving the following expression :—

$$\frac{15{,}833{\cdot}32\,(1{\cdot}016^{20}-1)}{0{\cdot}032+0{\cdot}08\,(1{\cdot}016^{20}-1)}=\frac{15{,}833{\cdot}32\times 0{\cdot}3736}{0{\cdot}032+0{\cdot}08\times 0{\cdot}3736}.$$

The calculation is as under :—

			0·3736
Log 0·3736	=	$\bar{1}$·57241	0·08
Log 15833·32	=	4·19957	0·029888
		3·77198	+ 0·032
−Log 0·06189	=	−$\bar{2}$·79162	0·06189
	Log^{-1}	4·98036	= £95,579.

For the present value of the royalties corresponding to the second decade, which are deferred for ten years, Formula 22 must be used, resulting in the expression :—

$$\frac{9{,}895{\cdot}82\ (1{\cdot}017^{20} - 1)}{0{\cdot}034 + 0{\cdot}08\ (1{\cdot}017^{40} - 1)}.$$

The calculation is as follows :—

$$\begin{aligned}
\text{Log } 1{\cdot}017 &= 0{\cdot}007321 \\
&\quad\ 40 \\
\text{Log}^{-1}\ 0{\cdot}29284 &= 1{\cdot}9626 \\
&\quad -1 \\
&\quad\ 0{\cdot}9626 \\
&\quad\ 0{\cdot}08 \\
&\quad\ 0{\cdot}077008 \\
&\quad +0{\cdot}034 \\
&\quad\ 0{\cdot}111008 \\
\text{Log } 1{\cdot}017 &= 0{\cdot}007321 \\
&\quad\ 20 \\
\text{Log}^{-1}\ 0{\cdot}14642 &= 1{\cdot}40093 \\
&\quad -1 \\
&\quad\ 0{\cdot}40093 \\
\text{Log } 9895{\cdot}82 &= 3{\cdot}99545 \\
\text{Log } 0{\cdot}40093 &= \bar{1}{\cdot}60307 \\
&\quad\ 3{\cdot}59852 \\
-\text{Log } 0{\cdot}111008 &= -\bar{1}{\cdot}04535 \\
\text{Log}^{-1}\ 4{\cdot}55317 &= £35{,}742.
\end{aligned}$$

The present value of the mineral property is, therefore,

$$£95,579 + 35,742 = £131,321,$$

and as the probable error in thickness has been seen to be 5 per cent., as obtained from the borings, the value may be written

$$£131,321 \pm 6,566.$$

Furthermore, the iron works may like to know how they stand in respect of their debenture issue. Let it be assumed that the property is ultimately purchased for £140,000, and that the necessary plant and equipment are estimated to cost £30,000; accordingly, the iron works propose to cover the whole of the outlay in respect of the mineral property by an issue of debentures to bring in something above £170,000. It is proposed to issue 6 per cent. debentures at 98, and underwriting, commission, brokerage, etc., are to cost 3 per cent., so that the iron works will get £95 for every £100 of debentures issued. Accordingly, the debenture issue is fixed at £180,000. Interest is to be paid, and the debentures are to be redeemed half-yearly, so that

$$n_1 = 10, \quad x = 2, \quad \text{and} \quad i_1 = 0{\cdot}06.$$

In order to redeem the debentures within the ten years and pay 6 per cent. upon the outstanding portion of the capital, it will be necessary to set aside an annual sum, which can be calculated in accordance with Formula No. 9, leading to the expression for the annual sum:—

$$\frac{180,000 \times 0{\cdot}06\,(1{\cdot}03^{20})}{1{\cdot}03^{20} - 1} = \frac{10,800 \times 1{\cdot}03^{20}}{1{\cdot}03^{20} - 1}.$$

The calculation is as follows :—

$$\begin{aligned}
\text{Log } 1{\cdot}03 &= 0{\cdot}128372 \\
&\quad\ 20 \\
\text{Log}^{-1}\ 0{\cdot}256744 &= 1{\cdot}8061 \\
&\quad\ -1 \\
&\quad\ 0{\cdot}8061 \\
\text{Log } 1{\cdot}03^{20} &= 0{\cdot}25674 \\
\text{Log } 10800 &= 4{\cdot}03342 \\
&\quad\ 4{\cdot}29016 \\
-\text{Log } 0{\cdot}086 &= \bar{1}{\cdot}90639 \\
\text{Log}^{-1}\ 4{\cdot}38377 &= £24{,}197.
\end{aligned}$$

This, then, is the sum that must be set aside yearly in two instalments for the service of the debentures. Towards this sum there is in the first instance the annual value of the royalty, or £15,833, but, in addition, there is the profit due to the mining operations ; it is true that there will be no profit in the strict sense of the word, but the iron works will effect a saving equivalent to profit by the difference between the market price that they would have to pay for the ore and what the ore will cost them to produce, which is over and above the value of the royalty. This difference is estimated at 1s. per ton, or £25,000 per annum ; the iron works, therefore, may reasonably expect to make a good profit on the entire transaction, seeing that they are entitled to expect the equivalent of an income of £40,833, out of which they have to pay £24,197 for the service of the debentures, leaving a balance of £16,636, which

will appear as additional profits realised by the iron works.

Finally, it may be desirable to draw up a debenture redemption table to show what proportion of the debentures is to be redeemed at the end of each half-yearly period. This table is calculated by the methods given on p. 189, and will be as follows, the amount of debentures redeemed being rounded off to an even number of pounds in each case :—

Period.	Interest.	Redemption.	Outstanding Capital.
1, ...	5,400	6,699	173,301
2, ...	5,199·03	6,900	166,401
3, ...	4,992·03	7,107	159,294
4, ...	4,778·82	7,320	151,974
5, ...	4,559·22	7,540	144,434
6, ...	4,333·02	7,766	136,668
7, ...	4,100·04	7,999	128,669
8, ...	3,860·07	8,239	120,430
9, ...	3,612·90	8,486	111,944
10, ...	3,358·32	8,741	103,203
11, ...	3,096·09	9,003	94,200
12, ...	2,826·00	9,273	84,927
13, ...	2,547·81	9,551	75,376
14, ...	2,261·28	9,838	65,538
15, ...	1,966·14	10,131	55,704
16, ...	1,662·21	10,437	44,970
17, ...	1,349·10	10,750	35,220
18, ...	1,026·60	11,072	23,148
19, ...	694·44	11,404	11,744
20, ...	352·39	11,746	..

The inference to be drawn is that, although the valuation has been performed quite equitably, and the vendor of the property gets as much for it as he is reasonably entitled to expect, the iron works would, nevertheless, be in the position, if necessary, to pay a considerably higher sum than the figure given by the

valuation, and yet stand to do well on the transaction, one of the main reasons for this position being that the iron works themselves are in so sound a condition that they are able to borrow the money they require on advantageous terms. This example affords a further illustration of the statement repeatedly made in the course of this work that mineral valuation is useful mainly for preparing a sound basis for negotiations, and that the price paid for a mineral property will in the last resort always be settled by bargaining between the parties concerned.

CHAPTER VII.

VALUATIONS FOR SPECIAL PURPOSES.

VALUATION FOR RATING PURPOSES.

A VALUATION of mines for rating purposes is occasionally called for; it must be made clear at the outset that the underlying principles are entirely different from those considered hitherto, the objects and methods being quite distinct from those described previously as those of a true equitable valuation. It may be noted in the first place that the rates in question are levied, not upon the mine, but upon the person in occupation thereof, although the rate is necessarily based upon the value assigned to the property. The basal principles are well given in a report by Sir Edward Hamilton and Sir George Murray *:—"All rates are charged, with a few exceptions, upon occupiers. . . . All properties which cannot be moved and which thus admit of being localised are liable to be assessed to local rates. . . . The basis of valuation of rateable property is annual value; that is to say, the rent which a hypothetical tenant may reasonably be expected to give for the privilege of occupying the property."

Originally, the only rate levied was the Poor Rate, the authority for the raising of which is contained in the well-known Statute 43 Eliz., C. 2, "An Act for the Relief of the Poor," which creates overseers of the poor, and empowers them to levy taxes upon every occupier of lands . . . coal mines, . . . etc. This statute

* Royal Commission on Local Taxation. Final Report, 1901.

has since been extended, and other rates have been added by a number of subsequent Acts of Parliament, the more relevant of which may be briefly referred to. The County Rates Act, 1852, 15 and 16 Vict., C. 81, consolidates and amends a number of different statutes, and practically assimilates the valuation for all local rates, such as county rate, highway rate, etc., to that for the poor rate, although it expressly says that the valuation for county rate is not bound to follow the poor rate valuation. It may be noted that in the case of mines the rateable value upon which poor rate is levied, is invariably the same as the assessable value upon which the general district rate is based; in urban districts these values differ for certain other forms of property—*e.g.*, agricultural land and railways—though they are the same for all forms in rural districts. In respect of valuation, perhaps the most important clause in the Act is that which states that: "the Words 'full and fair annual Value' shall be taken to mean the net annual Value of any property as the same is or may be required by law to be estimated for the purpose of assessing the Rates for the Relief of the Poor."

The Union Assessment Committee Act, 1862 (25 and 26 Vict., C. 103), together with the Acts of 1864 and 1880,* settled the machinery of valuation and provided for the appointment of assessment committees throughout the country.

The Rating Act, 1874 (37 and 38 Vict., C. 54) enacts that the Poor Rate Acts shall extend to "mines of every kind" not mentioned in the original Act of Queen Elizabeth, and makes the following provision with respect to mines:—

"Where a tin, lead or copper mine is occupied under

* Union Assessment Committee Assessment Act, 1864, 27 and 28 Vict., C. 39: Union Assessment Act, 1880, 43 and 44 Vict., C. 7.

a lease or leases granted without fine or a reservation wholly or partly of dues or rent, the gross value of the mine shall be taken to be the annual amount of the whole of the dues payable in respect thereof during the year ending on the thirty-first day of December preceding the date at which the Valuation List is made, in addition to the annual amount of any fixed rent reserved for the same, which may not be paid or satisfied by such dues.

"The rateable annual value of such mine shall be the same as the gross annual value thereof, except that where the person receiving the dues or rent is liable for repairs, insurance, or other expenses necessary to maintain the mine in a state to command the annual amount of dues or rent, the average annual cost of the repairs, insurance, or other expenses for which he is so liable shall be deducted from the gross value for the purpose of calculating the rateable value."

"The term 'mine' when a mine is occupied under a lease includes the underground workings and the engines, machinery, workshops, tramways, and other plant, buildings (not being dwelling-houses) and works and surface of land occupied in connection with and for the purposes of the mine, and situate within the boundaries of the land comprised in the lease or leases under which the dues or dues and rent are payable or reserved."

"After the commencement of this Act, the hereditaments to which the Poor Rate Acts are extended by this Act and which are thus made rateable to the relief of the Poor shall be rateable to all local rates in like manner as if the Poor Rate Acts had always extended to such hereditaments."

It has been shown that the method of arriving at the rateable value of tin, lead or copper mines is, there-

fore, quite clearly defined, being the royalty payable in the year preceding the valuation less the deductions indicated, and in respect of these mines the valuation for rating purposes presents no difficulty. But with regard to all other mines, including collieries and also quarries, the only indication given is that the gross estimated annual rental shall be the rent at which the hereditaments might reasonably be expected to let from year to year free of all the usual tenant's rates, etc., and no directions are given as to the manner in which this hypothetical rental is to be ascertained ; nor is the ascertainment rendered any easier by the insertion in the Act of the words " from year to year," since, as has often been pointed out, mines are not let in such manner. The result is that entirely different methods of rating are in vogue all over the country with widely different results ; a number of these were quoted by Sir Thomas Ratcliffe Ellis in his evidence given before the Royal Commission on Local Taxation,* and in a paper on the subject Mr. A. Hassam † points out that there were at least sixteen different methods of assessment in use in England and Wales in 1890, and there is no reason to suppose that any important unification of methods has been introduced since that date. In quite a number of instances the method indicated by the Act above quoted for tin, lead or copper mines has been followed, and the mines are assessed on the output or presumed output multiplied by the royalty per ton, in some cases with certain additions, in other cases with certain deductions ; in the case of collieries there is a complication even in this apparently straight-forward method, collieries in

* Minutes of Evidence taken before the Royal Commission on Local Taxation. Vol. i., 1898, Appendix Part 2, p. 68.

† "Taxation of Collieries." *Trans. Inst. Min. Eng.*, vol. xxix., 1905, p. 90.

some districts being assessed on the vend and in others on the total output without allowances for colliery consumption. In some districts the basis is the tonnage, but instead of the royalty an agreed rate per ton is used in arriving at the assessment, this agreed rate being an arbitrary one, apparently irrespective of the royalty, though no doubt often related to it. In a relatively few cases the profits realised by the colliery are used as the basis of valuation. The latter was the principle successfully contended for in the well-known case of the Denaby and Cadeby Collieries Company *v.* the Doncaster Union, 1898, when it was held that this was an admissible method of estimating the hypothetical rental, although it was never laid down that it was the only method that could legitimately be employed. The point is well taken by Mr. Walter C. Ryde, K.C.,* who writes that "as a matter of law profits must be regarded as affecting rateable value, just as far as those profits would, as a matter of fact, affect the rent that may reasonably be expected." Sir Edward Boyle † is a strong advocate for this method of valuation, and in his opinion the best method "would be to value the surface works and plant, in the same way as stations and other not directly productive works of a railway are valued. This value would be properly apportioned if in more than one parish. It would then be necessary to find the gross receipts from coal brought to bank, excluding engine coal, and from this sum to deduct all the expenses incurred in cutting the coal and bringing it to the surface for delivery. This would include not only miners' wages and all the underground expenses, but all the surface charges necessary to carry on the

* Ryde on "Rating." Fourth edition, p. 220.

† "The Principles of Rating," by Edward Boyle and G. Humphreys-Davies. Second edition, 1895, p. 517.

concern and get the coal realised. It would also include the repair of the tenant's plant and sufficient allowance for the depreciation by wear and tear of the same and its renewal. With all these sums deducted, the net income would be the sum left. From the latter sum should be deducted the amount which would be required for the remuneration of the tenant for his outlay of capital, skill, and risk. To ascertain this, it would be necessary to estimate the capital which a tenant would require for his non-rateable plant, such as horses, trams, tools, stores, floating capital for wages, etc., and on this amount he would be entitled to a return for interest and for trade profits. In the case of railways, as has been seen, it is usual to allow the tenant two and a half per cent. for risks and casualties, but manifestly a colliery is a much more hazardous undertaking, and a much larger amount should be allowed. Deducting this from the net income, we should have the gross annual value of the mine, and from this again would have to be deducted the average annual amount of the repairs, insurance, renewals, and 'other expenses necessary to maintain the hereditament in a state to command the rent'; we should then have the balance as the net rateable value."

It may be noted that, in common with most authorities, Sir Edward Boyle excludes colliery consumption in determining the profits. The author holds that this is wrong in principle, though it leads to a correct result. He contends that the value of the whole of the coal brought to bank, after being duly cleaned in case of need, should be credited to the colliery at the selling price which it fetches; as much of the coal as is used for colliery consumption should be charged among the disbursements under the head of stores consumed, being, of course, debited at the same price as that allowed for

its selling price; if it is an inferior coal to that which is marketed a correspondingly low price will be allowed. It is true that the same figure will thus appear on both the credit and the debit side of the account, so that the final result will not be affected, but the author's method does show the whole of the cost incurred in raising the coal, and makes comparison possible between a colliery that uses a portion of its own coal for colliery consumption and one that sells the whole of its raisings and buys any coal that it may need for its own purposes, or is completely electrified, and purchases the electric power which it requires.*

If the method advocated by Sir Edward Boyle is adopted, the rateable value of a colliery would be obtained as follows:—

From the sum realised by the sale of the coal deduct all working costs, both underground and above ground, expenses of office and administration, marketing expenses, miscellaneous disbursements, rates and taxes, the difference being the net income of the colliery.

From this income deduct the share of the income due to the occupier—namely, interest on the capital invested, trade profits at a reasonable scale, and an allowance for risks and casualties; the difference is the gross rateable value of the colliery.

From this gross value deduct the cost of repairs and maintenance (when not included under working costs), sinking fund for renewals, and insurance; the difference is the net rateable value of the entire undertaking. If the surface plant, buildings, land, railway lines, etc., are assessed separately, their rateable value should be deducted in order to arrive at the rateable value of the underground workings.

* See the author's paper on the "Economics of Coal Production." *Fuel Economy Review*, vol. i., No. 2, August, 1921, p. 10.

If the surface plant, etc., is situated in one parish, whilst the underground workings are in another or other parishes, the surface plant will, generally speaking, be wholly rated in the first-named parish, whilst the rateable value of the underground workings will be distributed amongst the parishes under which these lie in some agreed proportion, generally in the proportion of the tonnage raised or the acreage worked out from beneath each of the parishes in question.

The First Report of the Royal Commission on Local Taxation, 1899, p. 53, states the point thus :—" Where minerals are mined in one parish and brought to bank in another, they are assessed in the parish where they are mined, less shaft rent, etc., due in the parish where they are brought to bank, but where minerals are below low-water mark, and consequently in no parish, that portion of the mine is not rateable." The Commissioners " recommend that all mines or portions of mines of all classes worked under the sea below low-water mark should be subject to rates, and that the rates should be payable in the parish where the minerals are brought to bank," but this recommendation has not so far been carried into effect.

The above method of rating is, however, far from being generally accepted as the most satisfactory, and it was opposed strongly by Sir Thomas Ratcliffe Ellis on behalf of the Mining Association of Great Britain in giving evidence before the Royal Commission on Local Taxation already referred to. He is strongly opposed to taxation on profits, and advocates the method laid down by law in respect of tin, lead, and copper mines. His views are well indicated in the following passages from the memorandum which he presented :—

" The lettable value of the coal, in other words

the obtainable royalty, is the only safe basis for ascertaining the actual value."

"That value when ascertained should include the surface and underground workings, and the engines, machinery, workshops, tramways, and other plant (not being dwelling-houses) and works and area of land occupied in connection with and for the purposes of the mine."

"The mineral itself, unsevered, could have no rateable value. It is only when severed and brought to the surface that there can be any beneficial occupation of it that would make it rateable and the plant which is necessarily used for the purpose of giving to the mineral a rateable value should be included in the lettable value of the coal."

He further argues that collieries should be rated, not on their full annual value, but only on one-fourth of their annual value, because according to him the colliery is unable to benefit by a very large portion of the expenditure of the rates levied upon it. He gives interesting figures showing that the rateable value of all the coal mines in England and Wales according to the Poor Rate valuation in force on Ladyday, 1890, was £3,085,858, and that the coal output for the year 1889 was 153,596,360 tons, so that the average rateable value amounted to 4·82d. per ton, ranging in various counties from 1·42d. to 13·48d., the latter being an abnormally high figure, referring only to a very small quantity of coal. To-day, the average for the country is at least double what it was in 1890.

It will accordingly be seen that, not only does the actual practice of rating valuation vary very widely in different parts of the country, but the leading authorities on the subject also differ absolutely from each other as to what is the best way of valuing for rating

purposes, supposing that some scheme of unification were to be adopted. In principle, it would appear that a system based on profits ought to be the one most in agreement with the spirit of the law, because it is reasonable to suppose that the rental which a hypothetical lessee would give for a colliery must bear some relation to the profit that he could make out of it, whereas it is by no means certain that royalties afford an equally correct index. It might easily happen that a colliery working a very high-class coal on which a high royalty is paid might not be able to make as large profits as a colliery working inferior coal on which a lower royalty is charged, but it would appear logical to suppose that the latter colliery would command a higher rental than the first-named.

The main discussion of valuation for rating necessarily centres about the Poor Rate, not only because this is generally by far the largest item in the rates, but also because it was the first to be instituted, and the method of assessing for all other rates has been more or less assimilated to it; with respect to these other rates, it can, therefore, practically be taken for granted that they are based on the same principles or lack of principles as those which govern assessment for Poor Rate.*

In this connection the laws regulating the taxation of mines in several of the United States are interesting. For example, the laws of Wisconsin declare that "all mineral-bearing lands shall be annually assessed at the 'full cash value' which could be obtained therefor at a private sale." † "This 'full cash value' is described

* Since the above was written the Ministry of Health has been preparing a Bill intended to reform the chaotic situation here described.

† Wisconsin Geological and Natural History Survey, *Bulletin* xli., "A Study of the Methods of Mine Valuation and Assessment," by W. L. Uglow, p. 20.

as being 'the present worth at a certain rate of interest of the operating profits of a mine.'" * A similar law obtains in Michigan. Land in Pennsylvania (which includes coal and especially coal mines) is subject to taxation according to the actual value thereof, and to such rates and prices as the same would bring at a *bona fide* sale after due notice. It is interesting to note that the former law definitely stipulates for a private sale, whilst the latter contemplates a public one, but in neither case is any indication given as to how one or other is likely to affect the value. In some of the States—*e.g.*, Wyoming, South Carolina, and Nevada—it is definitely laid down that the products or proceeds of the mines shall be assessed and taxed.

It is hardly possible to say anything definite by way of guidance in rating valuations, having regard to the wide diversity in rating practice. It would indeed seem as though the valuer were in effect bound to conform to the practice of the particular assessment committee by which the mine falls to be rated, unless he is able to show that owing to the peculiar conditions of any one mine grave injustice would ensue, and even then it is by no means certain that he would obtain relief. In contradistinction to the ordinary equitable valuation of mineral properties, the valuer is here dealing with fairly well-ascertained facts, considerations of probability are practically excluded, and by the very terms of the Acts regulating the matter there is never any need to determine the present value of future revenues, because the Act definitely confines consideration to the year in which the rating takes place. The actuarial work is thus of the simplest, but, on the other hand, the evaluation of the various items entering into the complete assessment calls for considerable engineering experience,

* *Ibid.*, p. 31.

and, above all, for an intimate knowledge of the local conditions.

VALUATION FOR DEATH DUTIES.

It quite often happens that a valuer is called upon to determine the value of mines or mineral properties for the purpose of legacy or succession duty and estate duty. The principal Acts of Parliament that regulate the former of these duties are, firstly, the Stamp Act, 1815, 55 Geo. III., C. 184, which imposes stamp duties on probates of wills, legacies, and successions; it has, however, subsequently been greatly modified, first, by the Succession Duty Act of 1853 (16 and 17 Vict., C. 51), which states that "the interest of every successor . . . in Real Property shall be considered to be of the Value of an Annuity equal to the annual Value of such Property, after making . . . allowances, . . . and every such annuity for the purposes of this Act shall be valued according to the Tables in the Schedule annexed to this Act." (The schedule in question is in effect a table of the expectation of life at any given age.) "The yearly value of any . . . opened mine shall be calculated upon the average profits or income derived therefrom, after deducting all necessary outgoings during such a number of preceding years as shall be agreed upon for this purpose."

These duties have again been still further altered by the Customs and Inland Revenue Act, 1888 (51 Vict., C. 8), which declared that legacies arising out of real estate shall no longer be charged with Legacy Duties, so that they become liable to Succession Duty. Further modifications were brought about by the Finance Act, 1894 (57 and 58 Vict., C. 30), which made Succession Duty chargeable upon the principal value of the property after deducting Estate Duty, and proceeds to define the

principal value of real estate for the purpose of this Duty as being the same as for the purpose of Estate Duty, so that the definition of the latter lays down the governing principle for Death Duty valuations.

Estate Duty is levied in accordance with the Finance Act, 1894, 57 and 58 Vict., C. 30, by virtue of which Estate Duty is to be levied upon the principal value of all property, real or personal, which passes at death. The principal value is defined in the following terms :— "The principal value of any property shall be estimated to be the price that such property would fetch if sold in the open market at the time of the death of the deceased."

The same language is repeated in the Finance Act, 1910 (10 Edw. VII., C. 8), which again enacts that "In estimating the principal value of any property . . . the Commissioners shall fix the price of the property according to the market price at the time of the death of the deceased." The whole subject of death duties is one of exceptional intricacy, and must be left to lawyers conversant with it ; as far as valuation is concerned, it is fortunately unnecessary to consider these complexities, and indeed it may well be argued that a valuer will be the better able to discharge his duties if he does not know what rates are chargeable in any specific case, as it is certainly possible that such knowledge might unconsciously influence his valuation. His duty is limited to the latter task, and the only guidance that the law affords him is contained in the definitions of value above cited ; it has already been shown that there is no such thing as a market price for mineral properties in the ordinary sense of that phrase. Its meaning in the above Acts appears to be accepted as being the price that a willing buyer would give to a willing seller, so that, in other words, the essential work of the valuer is simply the determination of a present value that shall be the

equivalent, at the time of death, of the expected annual profits, as the selling price must be decided on this basis. This is thus a valuation on precisely the same lines as have been previously considered, and identically the same principles must apply, the method of calculation and the formulas to be used being those already indicated. Here, again, as in ordinary equitable valuations, the gravest difficulty will be the settlement of suitable rates of interest, and, in doing this, regard must be had, as already indicated, to the risk attending the enterprise.

The object of the valuation may either be mines in full work, minerals proved, developed, and leased, minerals proved but not developed, or land which there are good grounds for supposing to be mineral-bearing, and each of these four types must be considered separately.

In the case of mines, whether worked by the owner of the minerals or not, it has already been shown that the basis of the valuation must be the annual profit resulting from the mining operations, such profit being inclusive of the value of the unsevered minerals if the mines are worked by the owner of the minerals, whereas if the minerals are leased the profits in question are ascertained after the royalty rent, wayleaves, etc., and other charges have been deducted. The rate of interest to be allowed will necessarily be a relatively high risk rate owing to the hazardous nature of mining enterprise. Where the object of the valuation consists of minerals, developed and under lease, the valuation consists in determining the actual value of the expected royalty, based upon the output which may reasonably be expected. Dead rent must be taken into consideration, and the length of time the lease has to run will have to be considered. The risk rate in this case is a relatively

low one, especially if the mine has been producing steadily for a number of years already, and if the mineral deposit has been well proved and the existence of ample reserves has been ascertained. Any rights of relinquishment during the course of the lease that have been granted to a lessee must be taken into account, inasmuch as the existence of such a right and the possibility of its being exercised detract to some extent from the value of the lease. A lease for twenty-one years certain to a responsible lessee is evidently a more certain source of income, and, therefore, more valuable than a similar lease that can be broken at the end of every third year, even though the difference in value may not be readily quantifiable. The valuer must consider what the probability is that the mine may cease to yield profits at any time within the term of the lease, this being the only cause likely to lead to relinquishment, and should use the figure obtained by deducting such probability from unity as a fractional "weight" in determining the value of the lease. After the termination of the lease future royalties may have to be valued at a somewhat higher risk rate according to the probability that the lessees may or may not be prepared to renew, or that other lessees may be prepared to take up a lease upon approximately the same royalty rental. The possibility of diminished or enhanced royalties may have to be considered, and each case will require special treatment in accordance with the attendant circumstances.

Minerals that are proved but not developed offer somewhat greater difficulties, because there are fewer facts available upon which to base a valuation. It is quite possible that the proving may be sufficient to enable a reasonably close estimate to be formed as to the quantity and quality of the minerals; in this case the valuer will have to consider all the circumstances

of the case, the cost of opening out and developing the mineral property, the scale upon which it is capable of being worked, due regard being had to the probable capability of the markets for absorbing the products, and the probable profits to be realised by the mining operations, and upon such considerations he will probably be able to arrive at a reasonable royalty rent. If other similar properties in the same district are being worked, his task will be in so far simplified that the royalties charged by these will afford good guidance as to what the royalty should be. He will then be able to determine the present value of the expected annual rental; he will have to take into account that this will be a deferred value, because it will probably take a certain number of years before a suitable lessee for the property can be found; in the same way the risk rate of interest to be allowed would be somewhat higher than in the case of a property under lease, because it is a matter of doubt what royalty the lessor may be able ultimately to obtain.

Finally, the value of the estate, in which the existence of minerals is only suspected and not proved, can scarcely be said to be thereby increased. If, for instance, a property lies near the edge of a known coalfield, and from geological indications it is considered possible that the coal may extend under the estate, it is possible that some mining speculators might be prepared to pay a small amount for this possibility; it would never be large, because in most cases the cost of proving would be quite considerable. In the case of mineral veins of which there are surface indications, but no evidence as to value or persistence in depth, the usual plan for a mining speculator is to offer the landlord a small sum of money, usually only a few pounds a year, for a Take-note—that is, a licence to prospect—the licence to

carry with it the right to a mineral lease should prospecting operations be successful. Generally speaking, having regard to the fact that the value of the estate is deemed to be the price which it would fetch if sold in the open market, it cannot be said that the possibility of the existence of minerals hitherto unproved would add much, if anything, to the surface value. Cases have been known where a somewhat enhanced price has been given under such conditions by a mineral speculator, but the value attributable to the possibility of finding minerals is always extremely small. Here, again, of course, the strength and validity of the evidence in favour of the existence or otherwise of unproved mineral must be taken into account.

In much the same category may be placed the case of properties, on which the existence of mineral is either known or may be taken from geological evidence as being practically certain, but where the conditions of its occurrence are so unfavourable that it is unprofitable to work at present; possibly it may be worth working at some remote period, or again the chances are that it is never likely to prove workable. Under such conditions the existence of mineral cannot be said to add much to the value of the property. For example, in Northumberland there are considerable areas beneath which Limestone coals have either been proved to exist or may on geological grounds be looked upon as almost certainly existing. The seams are, however, often so thin and lie so deep that working them at present is out of the question, and in some cases it is quite doubtful whether their working will pay at any future time, whilst in other cases the situation is decidedly more favourable. It is hardly conceivable that anyone purchasing property underlain by seams of the former category would give any more for the property on

the remote chance that the coal may ever come into value.

VALUATION FOR LIQUIDATION.

It occasionally happens that after a mine has proved unsuccessful and has lost money, and has accordingly passed into the hands of a liquidator, it becomes necessary for the liquidator to attempt to sell the property, and in some cases he may find it necessary to have a valuation made. Such a valuation is always attended with considerable difficulty. The valuer starts with the knowledge that the venture has been an unsuccessful one ; he will have to determine in the first instance whether the failure is due to the intrinsic nature of the deposit itself, including its geographical position, or whether he can reasonably assume that defective methods of working or administration have been the real cause. In the former case, no value can be placed upon the mine as such, and the machinery and plant can be valued only at their removal value. In the latter case the valuer must determine what the probable annual profit may be expected to be if the mine is properly worked, and in that case he is entitled, not only to capitalise these profits, but also to value plant and machinery at site value as in a going concern. Very often he will come to the conclusion that, in order to bring the mine into a profit-earning condition, additional expenditure will be required either for development or for new plant, and possibly for new methods of treating the mineral product. He will have to bear in mind that the additional capital thus required to be raised will have to be invested in a venture already discredited by failure, and that, therefore, the conditions under which such capital can be obtained are likely to be far more onerous than would be the case

in a new venture. For the same reason he will probably find it necessary to assume a considerably higher risk factor than he would if he had been starting without the disastrous experience of the old concern behind him. On all these grounds the value of a property in liquidation will come out considerably lower than that of a corresponding untried mining proposition. Problems of very considerable difficulty occasionally present themselves; for example, the mine may have been worked under a lease giving power to make up shorts throughout the whole term of the lease, and during the existence of the liquidating company there have been short workings for a number of years. If, therefore, work is recommenced and the output exceed the tonnage equivalent to the dead rent, this excess tonnage will be free from royalty to the extent of the previously accumulated shorts. These accumulated shorts have, accordingly, some value; they represent a contingent asset, which will only be capable of being realised if the output exceeds a definite quantity. All that the valuer can do is to estimate the probability of this realisation and to put a value upon it, having regard both to the measure of probability of its attainment and to the probable time that must elapse before it is attained. It need hardly be said that great experience is needed in order to arrive at any reasonable figure in such a case as the above, and that different valuers are likely to differ very widely in their estimates. Generally speaking, valuers in dealing with a mine in liquidation find it advisable to exercise a very great measure of prudence and to make ample allowances for the risks which must be incurred.

VALUATION FOR LOANS.

It happens at times that money is borrowed in connection with mineral properties, and the mining engineer

is occasionally called upon to value the security on behalf of the lender. Mining companies, as a general rule, borrow money by way of debentures, and the valuer has in such cases to assure himself that the quantity of mineral existing in the mine and the probable profit obtainable by working it are capable of producing a sum sufficient to satisfy, not only the debenture interest, but also to provide for the redemption of the debenture capital.

The owner of mineral rights may occasionally desire to raise money on these by way of mortgage, and here again the task of the valuer is not a difficult one, being generally confined to an estimate of the probable quantity of mineral which forms the security for the mortgagee. The powers of a mortgagee of mineral rights vary considerably in different countries, and due regard must be paid to this point, although it hardly falls within the province of a valuer in the ordinary way.

Occasionally advances are made to a mining company by users of the mineral, the general arrangement being that the loan and interest thereon shall be repaid out of consignments of mineral sold by the mine to the lender of the money; the latter usually covenants to take a minimum quantity of mineral within a definite interval, and frequently also has a call upon the whole or a large portion of the output of the mine; frequently also he receives special treatment in the form of a rebate from the regular market price of the mineral to an extent agreed upon between the parties. In this case, again, the valuing engineer is concerned mainly with the question of quantity; such an arrangement would hardly be entered into unless both parties had confidence in each other and the lender knew the quality of the mineral, and had reason to suppose that the mine would be worked to best advantage.

Numerous modifications of these various forms of advance occur in practice, but, generally speaking, the mining engineer is called upon chiefly to determine that the probable quantity of mineral is sufficient to enable any bargain made to be carried out.

VALUATION FOR INVESTMENT.

In conclusion, attention may be drawn to the fact that a valuer is often called upon to advise as to the relative value of investments in mines and investments in other forms of security. The guiding principles in such cases are to bear in mind that mining is essentially speculative in its nature, and that on this account money invested in mining ventures must earn a risk rate of interest. This emphatically does not mean that the man who invests in mines gets upon the whole a larger return for his money than the man who invests in other securities; if this were the case, the price of mining investments would appreciate until the interest earned by the investment therein would be reduced to the same level as that obtainable from other securities. On the contrary, it means that if a man invests in mining securities, a certain number of his investments are, from the nature of the case, bound to turn out unsuccessful, and the object of the introduction of a risk rate is to compensate him for such insecurity. It has already been explained that the ratio of failures to successes constitutes the basis of what has here been called the risk factor. If an investor could invest in an infinite number of mines, the result ought to be the same as if he invested in any first class gilt-edged securities, the high interest that he would obtain in some cases compensating him for the total loss that he would sustain in others. For example, it has been repeatedly shown that the net income derivable from all the collieries

in the United Kingdom approximates very closely to the standard rate of interest on gilt-edged securities.*

Above all, the cardinal fact must not be lost sight of that a mine is necessarily always a wasting asset, and that revenue derived from a mine must not be looked upon as wholly income, but as being in part income and in part a sinking fund for the replacement of the capital invested. It matters nothing whether the investor in fact uses his revenue for the latter purpose or not, he must look upon it from this point of view in comparing a mining investment with those other investments in which his capital remains intact.

Furthermore, the effects of income tax, and, where applicable, mineral rights duty, must be borne in mind. It has already been shown how the incidence of these taxes operates as an additional disadvantage against the investor in mines or mineral property, and it is only when due consideration is given to all these points that it is possible to determine whether the rate of interest offered by a mining speculation, having regard to the wasting nature of the security, is a reasonable equivalent for the speculative risk that is incurred.

* See, *e.g.*, "Capital and Labour in Coal Mining," by John Bell Simpson, 1900, p. 24.

A = Actual value (at time of commencing operations).
P = Present value (at commencement of period of deferment).
a = Annuity or income payable at end of an accounting period.
i = Rate per unit of accumulative interest corresponding to accounting period.
r = Risk factor.
$s = r\,i$ = Rate per unit of risk interest corresponding to accounting period.
n = Number of accounting periods.
d = Number of periods of deferment.

1. Effective rate of interest per unit equivalent to i_1 :—

$$j = \left(1 + \frac{i_1}{x}\right)^x - 1 \text{ or } \left(1 + \frac{i_1}{x}\right)^x = j + 1.$$

2. Amount of a capital sum A earning risk rate of interest which accumulates at accumulative rate :— $\mathrm{S} = \mathrm{A} + \frac{\mathrm{A}\,s}{i}[(1+i)^n - 1]$.

3. Conversely :— $\mathrm{A} = \frac{\mathrm{S}\,i}{i + s[(1+i)^n - 1]}$.

4. Single-rate annuity :— $\mathrm{A} = \frac{a}{i}\left[\frac{(1+i)^n - 1}{(1+i)^n}\right] = \frac{a}{i}[1 - (1+i)^{-n}]$.

5. Annual amount required to redeem sinking fund S :— $a = \mathrm{S}\,i\,[(1+i)^n - 1]^{-1}$.

6. Single-rate annuity increasing in arithmetical progression by difference b :—

$$\mathrm{A} = a\left(\frac{(1+i)^n - 1}{i\,(1+i)^n}\right) + \frac{b}{i}\left(\frac{(1+i)^{n-1} - 1}{i\,(1+i)^{n-1}}\right) - \frac{(n-1)\,b}{i}\left(\frac{1}{(1+i)^n}\right).$$

7. Single-rate annuity increasing in geometrical progression by factor k :—

$$\mathrm{A} = \frac{a}{(1+i)^1}\left(\frac{k^n - (1+i)^n}{k - (1+i)}\right).$$

8. Single-rate annuity when periods for accumulative interest and for annuity payments are different :—

$$\mathrm{A} = \frac{a_1}{y}\left(\frac{\left(1 + \frac{i_1}{x}\right)^{n_1 x} - 1}{\left(1 + \frac{i_1}{x}\right)^{n_1 x}\left[\left(1 + \frac{i_1}{x}\right)^{\frac{x}{y}} - 1\right]}\right).$$

9. Single-rate annuity when both fractional periods of the year are the same, or $x = y$:—

$$\mathrm{A} = \frac{a_1}{i_1}\left(\frac{\left(1 + \frac{i_1}{x}\right)^{n_1 x} - 1}{\left(1 + \frac{i_1}{x}\right)^{n_1 x}}\right) = \frac{a_1}{i_1}\left[1 - \left(1 + \frac{i_1}{x}\right)^{-n_1 x}\right].$$

10. Two-rate annuity :— $\mathrm{A} = \frac{a\,[(1+i)^n - 1]}{i + s\,[(1+i)^n - 1]} = \frac{a}{s + i\,[(1+i)^n - 1]^{-1}}$.

11. Two-rate annuity when periods for accumulative interest and for annuity payments are different :—

$$\mathrm{A} = \frac{a_1\left[\left(1 + \frac{i_1}{x}\right)^{n_1 x} - 1\right]}{y\left[\left(1 + \frac{i_1}{x}\right)^{\frac{x}{y}} - 1\right] + s_1\left[\left(1 + \frac{i_1}{x}\right)^{n_1 x} - 1\right]}.$$

12. Two-rate annuity when both fractional periods of the year are the same, or $x = y$:—

$$\mathrm{A} = \frac{a_1\left[\left(1 + \frac{i_1}{x}\right)^{n_1 x} - 1\right]}{i_1 + s_1\left[\left(1 + \frac{i_1}{x}\right)^{n_1 x} - 1\right]}.$$

a_1 = Annuity or income payable annually.
i_1 = Annual rate of accumulative interest per unit.
s_1 = Annual rate of risk interest per unit.
n_1 = Number of operative years.
d_1 = Number of years of deferment.
x = Number of times in the year that accumulative interest is payable.
y = Number of times in the year that an annuity is payable.

13. To find a risk factor r applicable to the two-rate annuity which shall give the same result as the risk factor r_0 used for the single-rate annuity :—

$$r = \frac{r_0 (1 + i\, r_0)^n}{(1 + i\, r_0)^n - 1} - \frac{1}{(1 + i)^n - 1}.$$

14. Two-rate annuity increasing in arithmetical progression by difference b :—

$$A = \frac{a + \frac{b}{i}}{s + i\,[(1 + i)^n - 1]^{-1}} - \frac{n\,b}{i + s\,[(1 + i)^n - 1]}.$$

15. Two-rate annuity increasing in geometrical progression by factor k :—

$$A = a \left(\frac{(1 + i)^n - k^n}{(1 + i) - k}\right) \left(\frac{i}{i + s\,[(1 + i)^n - 1]}\right).$$

16. Single-rate deferred annuity :—$P = \frac{a}{i} \left(\frac{(1 + i)^n - 1}{(1 + i)^{n+d}}\right)$.

17. Single-rate deferred annuity when a different rate of interest i_0 is paid during deferment period :— $P = \frac{a}{i} \left(\frac{(1 + i)^n - 1}{(1 + i)^n (1 + i_0)^d}\right)$.

18. Single-rate deferred annuity when interest is paid at fractional periods of the year during deferment :— $P = \frac{A}{\left(1 + \frac{i_1}{x}\right)^{d_1 x}}$.

19. Two-rate deferred annuity when deferment period is an idle period :—

$$P = \frac{a}{(1 + i)^d} \left(\frac{1}{s + i\,[(1 + i)^n - 1]^{-1}}\right).$$

20. Two-rate deferred annuity when deferment period is an operative period :—

$$P = \frac{a\,[(1 + i)^n - 1]}{i + s\,[(1 + i)^{d+n} - 1]}.$$

21. Two-rate annuity deferred for an operative period when periods for accumulative interest and annuity payments are different :—

$$P = \frac{a_1 \left[\left(1 + \frac{i_1}{x}\right)^{n_1 x} - 1\right]}{y \left[\left(1 + \frac{i_1}{x}\right)^{\frac{x}{y}} - 1\right] + s_1 \left[\left(1 + \frac{i_1}{x}\right)^{(n_1 + d_1) x} - 1\right]}.$$

22. Two-rate annuity deferred for an operative period when both fractional periods of the year are the same, or $x = y$:—

$$P = \frac{a_1 \left[\left(1 + \frac{i_1}{x}\right)^{n_1 x} - 1\right]}{i_1 + s_1 \left[\left(1 + \frac{i_1}{x}\right)^{(n_1 + d_1) x} - 1\right]}.$$

23. Two-rate annuity deferred for an operative period, allowing for residual value of plant, assumed equal to P/c :—

$$P = \frac{a\,[(1 + i)^n - 1]}{\frac{c - 1}{c} \{i + s\,[(1 + i)^{d+n} - 1]\}}.$$

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